School of Mathematics & Statistics

Pure Mathematics Colloquium

On the connectivity of the escaping set in the punctured plane

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3 September 2019
15:30
Room 306 Alan Turing

Abstract:

A function f is a transcendental self-map of the punctured plane if $f:\mathbb{C}^*\to\mathbb{C}^*$ is a holomorphic function, $\mathbb{C}^*=\mathbb{C}\setminus\{0\}$, and both 0 and ∞ are essential singularities of f. For such maps, the escaping set $I(f)$ consists of the points whose orbit accumulates to a subset of $\{0,\infty\}$. We will look at the connectivity of $I(f)$ and show that either $I(f)$ is connected, or has infinitely many components. We also proved that $I(f)\cup \{0,\infty\}$ is either connected, or has exactly two components, one containing 0 and the other ∞. This gives a trichotomy regarding the connectivity of the sets $I(f)$ and $I(f)\cup \{0,\infty\}$, and we will give examples of functions for which each case arises. To give an example of a transcendental self-map f of $\C^*$ for which $I(f)$ is connected, we adapted the so-called spider's web structure due to Rippon and Stallard to the punctured plane. Finally, whereas Baker domains of transcendental entire functions are simply connected, we showed that Baker domains can be doubly connected in $\C^*$ by constructing the first such example. We also proved that if f has a doubly connected Baker domain, then its closure contains both 0 and ∞, and hence $I(f)\cup\{0,\infty\}$ is connected in this case. This is a joint work with Vasiliki Evdoridou (Open University) and Dave Sixsmith (University of Liverpool).

Tea and coffee will be available in the common area from 15:00