The Failing Firm Defence: Merger Policy and Entry*

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Abstract

This paper considers the 'failing firm defence'. Under this principle, found in most antitrust jurisdictions, a merger that would otherwise be blocked due to its adverse effect on competition is permitted when the firm to be acquired is a failing firm, and an alternative, less detrimental merger is unavailable. Competition authorities have shown considerable reluctance to accept the failing firm defence, and it has been successfully used in just a handful of cases. The paper considers the defence in a dynamic setting with uncertainty. A firm entering a market also considers its ease of exit, foreseeing that it may later wish to leave should market conditions deteriorate. By facilitating exit in times of financial distress, the failing firm defence may encourage entry sufficiently that welfare is increased overall. This view of the defence has several implications relevant to a number of merger cases. The conditions under which greater leniency is welfare-improving are examined.

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1. Introduction

Should a firm that is in financial distress be allowed to merge with a rival; should the 'failing firm defence' (FFD) be accepted as a general merger rule? If so, at what point should merger be allowed; that is, how lenient should merger policy be? Policy-makers have, so far, viewed the FFD with some suspicion: the conditions governing the application of the FFD are strict and it has been successfully used in just a handful of cases in which firms face the prospect of imminent bankruptcy. This paper presents a new view of the FFD, emphasizing its role in encouraging entry into a market. The analysis provides a framework for determining how lenient merger policy should be towards failing firms. It challenges current policy conclusions in a number of ways.

The FFD, in one form or other, is recognized by many countries. In the U.S., the defence is included specifically in the Department of Justice (DoJ) and Federal Trade Commission (FTC) 1992 Horizontal Merger Guidelines. In the European Union (EU), the provision for the defence is less explicit; the Commission's case law has developed, however, the concept of a 'rescue merger'.[1] Policy discussions of the FFD mergers[2] have reached three broad conclusions. First, in the absence of any other benefits (such as avoiding exit costs, or social benefits), these mergers should not be allowed if they increase market power.[3] Secondly, the failing firm should be genuinely failing, and not merely 'flailing'. This means that merger should be allowed only when the alternative is immediate bankruptcy; and the failing firm should not receive a significant share of the gains from merger—if it does, this should be interpreted as a signal that the firm is not failing. Thirdly (and related to the first conclusion), the greater the weight on consumer welfare (i.e., anti-competitive effects), the less favourably is a failing firm merger viewed by regulators.[4]

[1]Different countries impose different conditions on the defence. For example, the 1992 Merger Guidelines in the U.S. also require that the failing firm "has made unsuccessful good-faith efforts to elicit reasonable alternative offers of acquisition of the failing firm that would both keep its tangible and intangible assets in the relevant market and pose a less severe danger to competition than does the proposed merger". This is not a requirement in the E.U..

[2]See, for example, the OECD Competition Policy Roundtable in 1996, as well as the cases described below.

[3]In the words of the U.S. DoJ, "since a merger to monopoly is the classic kind of merger that would normally be prohibited, why are such monopolies suddenly acceptable if one of the parties is failing?".

[4]For example, the Australian position is "where anti-competitive effects are expected, a merger may nonetheless be permitted on wider ... grounds".

The failing firm defence has been applied in a number of mergers where one (or more) party is experiencing financial difficulties. In the U.S., three cases in particular have been important in the establishment and development of the defence. The FFD was first used in 1930 in the case of *International Shoe*'s acquisition of a financially troubled competitor. The principle was developed further in the case of *Citizen Publishing Co.*, when the Supreme Court rejected a merger with a distressed newspaper company and set out stringent conditions under which the defence would be accepted. Finally, in the *General Dynamics* case in 1974, the Supreme Court concluded that the acquisition of a coal mining company was acceptable even though it produced a company with a large market share in a concentrated industry. The company being acquired was not in immediate danger of bankruptcy, but was declining in profitability. This raised the possibility of a 'flailing firm defence': justifying a merger on the grounds that one of the firms, while not in imminent danger, is at least in a position of financial weakness.

In European merger control, the case of *Kali und Salz* and *Mitteldeutsche Kali* (MdK) in 1993 established the principle of the failing firm defence (Case No. IV/M.308, 1994). Following a 30% fall in demand in the potash (fertiliser) market over the preceding five years, *Mitteldeutsche Kali* was facing bankruptcy (it was surviving only due to support from the Treuhand, which could not be continued due to EC Treaty provisions on state aids). Despite the combined market share of 98%, the European Commission found that MdK's market share would most likely go to *Kali und Salz* and permitted the merger on failing firm grounds. A recent merger in the chemicals sector reinforced the principle. In 2001, *BASF* was permitted to acquire *Eurodiol* and *Pantochim*, which were both in receivership, although this would result in market shares in excess of 45% in a number of solvents markets. No other buyer could be found and the Commission found that absent the merger the resulting reduction in capacity was likely to result in supply shortages and higher prices.

The break-up of the failed accountancy practice of *Arthur Andersen* (AA) in 2002, in which the various national divisions were acquired by other "Big Four" accountancy firms, may also be viewed on failing firm grounds. There could be little doubt that AA was no longer viable as a global player; the issue was rather whether an orderly acquisition was preferable to fragmentation of the national practices. The takeover of *British Caledonian* by *British Airways* in 1987 was accepted by the (then) Monopolies and Mergers Commission

(MMC) on failing firm grounds, although some commentators would regard this merger as an example of the promotion of a "national champion." The case was complicated by the obstacles to foreign-owned bidders posed by existing airport slot ownership rules. In 1998 the joint venture (JV) between the cross-Channel ferry operators *P&O* and *Stena Line* was exempted from the provision on anti-competitive agreements of Article 85(1) of the EC Treaty. Although not in imminent danger of failing, the prospect of intense competition from the Channel Tunnel and the loss of revenues from the ending of duty free sales threatened the continuation of independent ferry operations. In reaching its decision, the European Commission discussed but decided to ignore the effects of the JV on local economies.[5]

However, competition authorities have in several cases shown some reluctance to accept the failing firm defence, preferring to let the firms fight it out and give consumers the benefit of low prices during the ensuing war of attrition. In the U.K., the proposed merger in 1997 between *Scottish Pride*, a failing dairy firm in Scotland, and *Robert Wiseman Dairies*, fell through and *Scottish Pride* went into receivership, due to the delay imposed on merger while a report by the MMC was considered by the Department of Trade and Industry (DTI). *Scottish Pride* was in clear financial distress; but the DTI was concerned about the merged firm's 80% share of the Scottish milk market.[6] Doubt has also been expressed in some cases as to whether the 'failing' firm was in fact failing. In the US, the *Detroit News* and *Detroit Free Press* reached a joint operating agreement (JOA) in 1988, in response to continued losses by both papers. In the JOA, the papers agreed to set prices jointly, but to retain independent editorial functions. A key obstacle to the approval of the JOA was the division of profits. The initial administrative law judge decided that the equal division proposed in the JOA indicated that neither firm was failing, and hence the FFD provision in the Newspaper Provision Act of 1970 could not apply. This decision was subsequently overturned by the Attorney General, but only after a delay of almost four years.[7] In the fertiliser sector, the proposed sale of *ICI*'s loss-making fertiliser division to *Kemira Oy* in 1990 was blocked by the MMC due to possible adverse competition effects of the merger, despite the recognition by

[5]See `http://www.ebusiness.com/news/stories/102/10151.html`.

[6]See `http://www.competition-commission.gov.uk/wise.htm`, `http://www.oft.gov.uk/html/trading/tr-arch/nws16-3.htm` and `http://www.ukbusinesspark.co.uk/bpfood97.htm`.

[7]See Kwoka and White (eds). (1999), case 1 for further details.

the MMC that *ICI* might exit the market in due course.[8] The strength of the parent company was something of an obstacle in this case, as the loss-making division could be supported by the parent for some time and exit was therefore not considered to be an immediate prospect.

There has been very little formal economic analysis of the failing firm defence. While the literature on mergers generally is very large (see Jacquemin and Slade (1989) for a survey), there are very few papers analysing the FFD specifically. The only exception that we have been able to find is Persson (2001), who analyses the welfare consequences of the FFD, concentrating on the *ex post* efficiency of sales of the failing firm's assets. He shows that the detailed provisions of the FFD do not ensure that the socially preferred buyer obtains the assets. In our model, merger policy is used as a means to encourage *ex ante* entry to an industry. Merger leads to a more concentrated market structure, and consequently lower consumer surplus and greater deadweight loss. But the possibility of merger in times of financial distress increases the expected profitability of operating in a market; this, in turn, increases the willingness of firms to enter the industry, reducing concentration and deadweight loss from market power in the long run.

We argue, therefore, that rescue mergers are desirable precisely because they increase firms' market power and so profits in times of financial distress. In effect, merger policy affects the sunkness of the entry decision and therefore the timing of entry. Entry occurs sooner when firms are allowed to merge and thus increase profit when one of the firms is failing. If the entrant is also likely to be the first to exit (because the incumbent has some intrinsic advantage, for example), then allowing the failing firm to gain a larger share from merger encourages entry. Finally, a lenient merger policy (allowing merger at an early stage of financial distress) may harm the incumbent more than it benefits the entrant. A consumerist social planner disregards this, and sets a lenient merger policy to encourage early entry. A social planner who considers industry profits sets policy more strictly.

Although our main focus is on the interaction between merger policy and entry decision, note that similar considerations arise with any *ex ante* decision made by a firm. For example, the decision to extend an existing product line, initiate a research and development project, or undertake an advertising campaign, could be analysed in a similar fashion. What matters

[8]See `http://www.competition-commission.org.uk/reports/293.htm`.

for the analysis is that the decision involves a sunk cost, and that the returns are uncertain and affected by the prospects of future merger. We have chosen entry as an important example of such a decision; but the analysis can be applied to other issues.

The lack of formal economic analysis of the FFD extends to empirical study. We cannot, therefore, provide any direct evidence concerning the empirical importance of our argument. We note, however, that the analysis here bears many resemblances to that of the effect of bankruptcy procedures on *ex ante* decisions by firms and shareholders.[9] There is growing literature on this question. Jensen and Meckling (1976) and Green (1984) argue that bankruptcy procedure can induce inefficient management decisions concerning investment, distribution of dividends and financing. Mooradian (1994) analyses the effect of bankruptcy protection on *ex ante* investment policy of managers. Bebchuk (2002) shows how deviations from absolute priority in bankruptcy proceedings can bias managers in favour of choosing riskier projects. Even with this recognition of the importance of the relationship between bankruptcy procedures and *ex ante* decisions, there has been, to our knowledge, little empirical work in the area. Fan and White (2002) is a notable exception. They examine whether individuals are more likely to become entrepreneurs if they live in states in the U.S. with higher bankruptcy exemptions.[10] They find that households are more likely to own and start businesses if they live in states with higher bankruptcy exemption levels. In summary: the extensive theoretical analysis, and rather more limited empirical study, of the relationship between bankruptcy and *ex ante* decisions lends weight to the likely relevance of our argument that merger policy for failing firms affects entry.

In section 2, we start with a simple two-period model to illustrate the trade-off between encouraging entry and increasing market power. In the remainder of the paper, we develop a multi-period (in fact, continuous-time) dynamic model of entry and exit in an industry when returns are uncertain and entry and exit involves irreversible decisions. (There are, therefore, 'real options' involved; see Dixit and Pindyck (1994) for an introduction.) There are three main objectives for the analysis. First, we provide an explicit determination of equilibrium entry, exit and merger decisions; this is done in sections 3 and 4. Secondly, we determine

[9] Many papers on bankruptcy procedures concentrate on *ex post* efficient division of bankruptcy value; see e.g., Hart (1995).

[10] Entrepreneurs filing for personal bankruptcy under Chapter 7 must give up all of their assets in excess of an exemption level to discharge debts.

analytically the conditions under which merger policy is effective in encouraging entry; see section 5. Thirdly, in section 6, we examine numerically the comparative statics of optimal merger policy, assessing, for example, how the degree of uncertainty about profitability of the industry affects the merger policy that should be adopted. Section 7 considers the importance of the market structure assumption used in the model; section 8 concludes. The appendix contains technical details and longer proofs.

2. The Model

2.1. An Example

A simple two-period, two-firm example illustrates some of the issues. One firm, the incumbent, operates in a market for both periods. The other firm chooses whether to enter or not at the beginning of the first period, before information about the profitability of the industry is revealed; and whether to exit at the beginning of the second period, after this information is received. The per-period variable profit from operating in the market is uncertain. For the incumbent as monopolist, it is equal to $\theta\pi(1)$; when both firms are in the market, both receive a per-period profit of $\theta\pi(2)$. The entrant faces a sunk entry cost $E > 0$. Per-period social surplus from the market is $\theta SS(1)$ when one firm operates, and $\theta SS(2)$ when both operate. Suppose that $SS(1) < SS(2)$ i.e., the deadweight loss through market power outweighs the entry cost. θ is a random variable, realized in the first period after the entrant's entry decision. The firms' common prior over θ is uniformly distributed over the interval $[0, 1]$. There is no discounting.

Consider first the entrant's decisions when the firms are not allowed to merge after entry. If it has entered, it earns a expected per-period variable profit of $\pi(2)/2$. Hence the entrant enters iff $\pi(2) > E$. To make the illustration as clear as possible, suppose that $\pi(2) < E$, so that entry is not privately optimal; it is, however, socially optimal, since we assume that $SS(2) \geq SS(1)$. The policy-maker can use merger policy to correct this inefficiency. Suppose that the firms are allowed to merge after information about the market is received, if the realization of θ is sufficiently low—below θ_M, say.

If the firms merge, suppose that the entrant receives an amount s^E from the surplus generated by the merger. (For example, it might receive a fraction $b \in [0,1]$ of the extra per-period profit.) Hence the entrant's profit in the second period, if merger occurs, is $\theta(\pi(2) + s^E)$. Therefore its expected profit from entry is

$$\pi(2) + \left(\frac{s^E}{2}\right)\theta_M^2 - E.$$

Hence the merger policy makes entry privately optimal when

$$\theta_M \geq \sqrt{\frac{2(E - \pi(2))}{s^E}} \equiv \underline{\theta_M}.$$

Expected social surplus from entry is

$$SS(2) - \left(\frac{SS(2) - SS(1)}{2}\right)\theta_M^2.$$

Therefore, this merger policy increases social welfare iff

$$\theta_M \leq \sqrt{\frac{2SS(2)}{SS(2) - SS(1)}} \equiv \overline{\theta_M}$$

(given that it is assumed that no entry occurs in the absence of the policy).

The policy can increase social welfare by encouraging the entrant to enter the market if $\overline{\theta_M} \geq \underline{\theta_M}$. If this is the case, then of course the policy-maker prefers the lowest possible level of θ_M. So in this simple two-period example, the optimal merger policy is given by the corner solution $\theta_M = \underline{\theta_M}$.

This simple example is useful to illustrate the basic message, but it suffers from several limitations. Most seriously, the optimal merger policy necessarily is determined by a corner solution. This misses various factors, since the trade-off determining the optimal policy is not continuous. In the more general setting considered next, the policy-maker faces a trade-off in determining the leniency of merger policy. A strict policy (low θ_M) increases the expected prevalence of competition, and hence expected welfare, when entry occurs; a lenient policy (high θ_M) encourages entry (by making post-entry expected profits greater) and so also increases competition and welfare.

Two risk neutral firms each can produce and sell in a market. One firm, the incumbent I, is in the market at the opening of the model and never exits; the other firm, the entrant E, decides when to enter and to exit. (This imposed asymmetry between the firms is to clarify the main issues for analysis; its implications are discussed in section 7.)

The timing of entry and exit of the entrant, and of merger between the two firms (when merger is permitted) is the main concern of the analysis. Time is continuous and labelled by $t \in [0, \infty)$. The decisions to enter, exit and merge can be delayed indefinitely. Once the entrant has exited, it can never again enter the market. This limits the analysis to one 'cycle' of entry and exit, for simplicity; further cycles could be considered without affecting the results qualitatively.

We use reduced-form functions for the flow payoffs that the firms receive when they operate in the market. If the incumbent I is the only firm in the market, then it receives $\theta_t \pi(1) > 0$; $\theta_t > 0$ is described further below. If both firms are operating in the market, then the incumbent receives a flow payoff of $\theta_t \pi(2) > 0$. The entrant receives a flow payoff of zero outside of the market, and $\theta_t \pi(2)$ after entry while in a duopoly. It pays an entry cost $E > 0$ upon entry, which is recoverable on exit.[11] It is assumed that $\pi(1) > 2\pi(2)$: the standard efficiency effect assumed in many industrial organization models. The firms bargain to divide the surplus from the merger, with a fraction $b \in [0, 1]$ given to the entrant and the remaining fraction $1 - b$ to the incumbent. (See section 4 for further details.) Consumer surplus is $\theta_t CS(1)$ when only one firm (either the incumbent or merged firm) operates in the market, and $\theta_t CS(2) > \theta_t CS(1)$ when two firms operate in the market.

θ_t is assumed to be exogenous and stochastic, evolving according to a geometric Brownian motion (GBM) with drift:

$$d\theta_t = \mu \theta_t dt + \sigma \theta_t dW_t \tag{1}$$

[11]Note that although E is not sunk, there is nevertheless irreversibility in the model, since the entrant is not allowed to re-enter the market once it has exited. The model could have been written with a sunk cost of entry (and exit) with similar results. This formulation is easier to work with.

where $\mu \in [0, r)$ is the drift parameter, measuring the expected growth rate of θ,[12] $\sigma > 0$ is the instantaneous standard deviation or volatility parameter, and dW_t is the increment of a standard Wiener process $\{W_t\}_{t \geq 0}$, so that $dW_t \sim N(0, dt)$. The parameters μ, σ and r are common knowledge and constant over time. The choice of continuous time and this representation of uncertainty is motivated by the analytical tractability of the value functions that result.

The strategies of the agents are now defined. First consider the case where merger is not permitted. If the entrant E has not entered at any time $\tau < t$, its action set is $A_t^E =$ {enter, don't enter}. If, on the other hand, firm E has entered at some $\tau < t$, then $A_t^E =$ {exit, don't exit}. The incumbent makes no moves. When merger is permitted, then the action sets are as follows. If the entrant E has not entered at any time $\tau < t$, its action set is $A_t^E =$ {enter, don't enter}. If it has entered at some $\tau < t$, then $A_t^E =$ {exit, don't exit and merge, don't exit and don't merge}. If the entrant has entered, then the incumbent's action set is $A_t^I =$ {merge, don't merge}. Merger can occur if and only if (i) it is permitted by the policy-maker; (ii) both firms agree to merge.[13]

A strategy for firm $i \in \{E, I\}$ is a mapping from the history of the game H_t (the sample path of the stochastic variable θ and the actions of both firms up to time t) to the action set A_t^i. Firms are assumed to use stationary Markovian strategies: actions depend on only the current state and the strategy formulation itself does not vary with time. Since θ follows a Markov process, Markovian strategies incorporate all payoff-relevant factors in this game. Furthermore, if one firm uses a Markovian strategy, then its rival has a best response that is Markovian as well. Hence, a Markovian equilibrium remains an equilibrium when history-dependent strategies are also permitted, although other non-Markovian equilibria may then also exist. (For further explanation see Maskin and Tirole (1988) and Fudenberg and Tirole (1991).)

Finally, we assume that the policy-maker is able to commit to a merger policy. In this

[12]The restriction that $\mu < r$ ensures that there is a positive opportunity cost to holding the 'option' to enter, so that the option is not held indefinitely.

[13]Given the way in which we model the bargaining over the surplus from merger, if there is positive surplus, then each firm automatically has an incentive to merge. We assume that managers do not decline to merge when there is a gain to the firms' owners from doing so i.e., we ignore any managerial incentives that might arise in the merger. There is thus no role for hostile bids.

model, this means that the policy-maker chooses at the outset a critical value θ_M at or below which the firms are permitted to merge, if they choose to do so. Once chosen, the critical value cannot be revised by the policy-maker.

The following parametric assumptions are made:

Assumption 1: *(a)* $\mathbb{E}_0\left[\int_0^\infty \exp(-rt)\theta_t\pi(2)dt\right] < E$.

(b) $\alpha + \beta < (\beta/(\beta-1))^\alpha$.

Part (a) of the assumption states that the initial value of the project is sufficiently low that immediate entry is not worthwhile. (The operator $\mathbb{E}_0$ denotes expectations conditional on information available at time $t = 0$.) Part (b) ensures that there exists a solution to the entrant's entry decision problem when mergers are not allowed. (α and β are parameters defined in equations (3)–(4) in section 3.)

3. Case 0: Entry and Exit when Merger is not Permitted

This section develops the no-merger benchmark. In this case, the entrant's (pure Markovian) strategy takes the form of two critical values or 'trigger points' for the exogenous variable θ: θ_{E0}, at which the firm enters; and θ_X, at which it exits.

Detailed derivations of the firms' value functions are contained in the appendix. There we show that the entrant's value function, V_{E0}, has three components, holding over different ranges of θ:

$$V_{E0} = \begin{cases} A_{E0}\theta^\beta & \text{before entry,} \\ \frac{\theta\pi(2)}{r-\mu} + B_{E0}\theta^{-\alpha} & \text{after entry, before exit,} \\ 0 & \text{after exit,} \end{cases} \tag{2}$$

where there is a cost $E > 0$ to entry that is recoverable on exit. Prior to entry, the entrant receives a flow payoff of zero and holds an option to enter. The term $A_{E0}\theta^\beta$ is the value of this option. On entering, the entrant receives a flow payoff of $\theta\pi(2)$; it also holds an option to exit, the value of which appears as $B_{E0}\theta^{-\alpha}$ in the value function. Finally, exit takes the entrant's flow payoff to zero, since there is no option to re-enter. α and β are constants that are the positive roots of two characteristic equations (see the appendix):

$$\alpha = \frac{1}{2}\left(-\left(1-\frac{2\mu}{\sigma^2}\right)+\sqrt{\left(1-\frac{2\mu}{\sigma^2}\right)^2+\frac{8r}{\sigma^2}}\right) > 0, \tag{3}$$

$$\beta = \frac{1}{2}\left(1-\frac{2\mu}{\sigma^2}+\sqrt{\left(1-\frac{2\mu}{\sigma^2}\right)^2+\frac{8r}{\sigma^2}}\right) > 1. \tag{4}$$

α is less (greater) than β if $2\mu/\sigma^2$ is less (greater) than 1. A_{E0} and B_{E0} are constants that are determined by boundary conditions.

The boundaries between the three regimes are given by the trigger point θ_{E0} and θ_X of the stochastic process such that continued delay (immediate entry) is optimal for $\theta < (\geq)\theta_{E0}$ (conditional on not having yet entered); and continued operation (immediate exit) is optimal for $\theta > (\leq)\theta_X$ (conditional on having entered but not yet exited). The optimal stopping time T_E is then defined as the first time that the stochastic process θ hits the interval $[\theta_{E0}, \infty)$ from below; T_X is the first time after T_E that the stochastic process θ hits the interval $[0, \theta_X]$ from above (having previously hit the interval $[\theta_{E0}, \infty)$).

By arbitrage, the critical values θ_{E0} and θ_X each must satisfy a value-matching condition; optimality requires a second condition, known as 'smooth-pasting', to be satisfied. (See Dixit and Pindyck (1994) for an explanation.) This condition requires the components of the entrant's value function to meet smoothly at θ_{E0} and θ_X with equal first derivatives. It is shown in the appendix that the value matching and smooth pasting conditions imply that the optimal entry point for the entrant is given by the non-linear equation

$$(\alpha+\beta)B_{E0}\theta_{E0}^{-\alpha} = \beta E - (\beta-1)\frac{\theta_{E0}\pi(2)}{r-\mu}. \tag{5}$$

Assumption 1 ensures that there is at least one well-defined solution to this equation. When there are two solutions, only the larger solution is relevant (as inspection of the value functions reveals).

The optimal exit point is

$$\theta_X = \left(\frac{\alpha}{\alpha+1}\right)\left(\frac{E}{\pi(2)}\right)(r-\mu). \tag{6}$$

It is straightforward to show that $\theta_X < \theta_{E0}$, given assumption 1. Notice that θ_X is decreasing in $\pi(2), \mu$ and σ,[14] and increasing in r and E. These comparative statics are quite standard; for example, uncertainty creates an option value and so (generally) delays irreversible investment, relative to the net present value rule. The greater the degree of uncertainty, the larger this delay, and so the lower is θ_X. (See Dixit and Pindyck (1994) for more explanation, and Mason and Weeds (2003) for an exception to this general intuition.)

The constants A_{E0} and B_{E0} are

$$A_{E0} = \left(\frac{\alpha+1}{\alpha+\beta}\right)\left(\frac{\theta_{E0}\pi(2)}{r-\mu} - \left(\frac{\alpha}{\alpha+1}\right)E\right)\theta_{E0}^{-\beta}, \tag{7}$$

$$B_{E0} = \frac{1}{\alpha}\left(\frac{\pi(2)}{r-\mu}\right)\theta_X^{\alpha+1} > 0. \tag{8}$$

In summary: the equilibrium strategy of the entrant is "enter at the first time that θ hits the interval $[\theta_{E0}, \infty)$; after entry, exit at the first time that θ hits the interval $[0, \theta_X]$" where θ_{E0} and θ_X are given by equations (5) and (6) respectively.

Equation (6) for the entrant's exit trigger point can be interpreted as an effective exit cost with an adjustment for uncertainty. Exit reduces the flow payoff by $\theta\pi(2)$ and recovers the cost E; hence the normalized cost of exit is $E/\pi(2)$. With an effective interest rate of $r-\mu$ (i.e., the actual interest rate r minus the expected proportional growth in the flow payoff μ), this gives an instantaneous cost of $(r-\mu)E/\pi(2)$. If a Marshallian rule were used for the exit decision, the trigger point would be simply this cost. But with uncertainty, irreversibility and the option to delay exit, the Marshallian trigger point must be adjusted downward by the factor $\alpha/(\alpha+1) < 1$. (A similar intuition applies to the entry trigger point θ_{E0}, although the argument and so equation (5) is complicated considerably by the subsequent option to exit.)

[14] The last comparative static follows from the fact that α is decreasing in σ.

The incumbent's value function V_{I0} has three components:

$$V_{I0} = \begin{cases} \frac{\theta\pi(1)}{r-\mu} + A_{I0}\theta^{\beta} & \text{before entry,} \\ \frac{\theta\pi(2)}{r-\mu} + B_{I0}\theta^{-\alpha} & \text{after entry,} \quad \text{before exit,} \\ \frac{\theta\pi(1)}{r-\mu} & \text{after exit.} \end{cases} \tag{9}$$

A_{I0} and B_{I0} are constants determined by boundary conditions. $A_{I0}\theta^{\beta}$ and $B_{I0}\theta^{-\alpha}$ are option-like terms that anticipate the actions of the entrant. Since the incumbent does not choose an action, the smooth pasting optimality condition does not apply. Value functions are forward-looking, however, and so value matching applies at θ_E and θ_X. These two conditions yield

$$A_{I0} = -\left(\frac{\pi(1)-\pi(2)}{r-\mu}\right)\left(1-\left(\frac{\theta_X}{\theta_{E0}}\right)^{\alpha+1}\right)\theta_{E0}^{-(\beta-1)} < 0, \tag{10}$$

$$B_{I0} = \left(\frac{\pi(1)-\pi(2)}{r-\mu}\right)\theta_X^{\alpha+1} > 0. \tag{11}$$

The social planner's value function $V_{S0}(\theta_t)$ at time t is a weighted sum of consumer surplus and firms' profits, with a weight $\lambda \in [0,1]$ attached to the latter: $V_{S0} = V_{C0} + \lambda(V_{I0} + V_{E0})$. V_{C0}, the value function of consumers, is

$$V_{C0} = \begin{cases} \frac{\theta CS(1)}{r-\mu} + A_{C0}\theta^{\beta} & \text{before entry,} \\ \frac{\theta CS(2)}{r-\mu} + B_{C0}\theta^{-\alpha} & \text{after entry,} \quad \text{before exit,} \\ \frac{\theta CS(1)}{r-\mu} & \text{after exit.} \end{cases} \tag{12}$$

Again, since the social planner does not choose an action, only value matching conditions apply at θ_{E0} and θ_X. The constants A_{C0} and B_{C0} are then determined as

$$A_{C0} = \left(\frac{CS(2)-CS(1)}{r-\mu}\right)\left(1-\left(\frac{\theta_X}{\theta_{E0}}\right)^{\alpha+1}\right)\theta_{E0}^{-(\beta-1)} > 0, \tag{13}$$

$$B_{C0} = -\left(\frac{CS(2)-CS(1)}{r-\mu}\right)\theta_X^{\alpha+1} < 0. \tag{14}$$

4. Entry and Exit when Merger is Permitted

Now suppose that the firms are permitted to merge, but only when the state variable is at a sufficiently low level—less than a critical value denoted θ_M. In this section, the merger point θ_M is treated as a parameter: it is determined by the policy-maker and is outside the firms' control. In the next section, we shall consider how the policy-maker chooses θ_M. This section therefore determines the firms' behaviour for all values of θ_M. We start by supposing that the firms choose to merge (at some level of the state variable); we then consider when this outcome will occur in equilibrium.

Once merged, the firms operate as a monopoly, earning a flow payoff of $\theta\pi(1)$ (i.e., monopoly profit). The entrant's entry cost E is recovered by the merged firm (for example, the cost relates to capacity that is not needed by the post-merger monopoly, and which is sold.) The alternative to merger is case 0: after entry, that exit occurs at θ_{E0}. Hence the value of the surplus to the firms from merger is

$$
\begin{aligned}
S_M(\theta) &= \frac{\theta\pi(1)}{r-\mu} + E - \left(\frac{\theta\pi(2)}{r-\mu} + B_{E0}\theta^{-\alpha}\right) - \left(\frac{\theta\pi(2)}{r-\mu} + B_{I0}\theta^{-\alpha}\right) \qquad (15)\\
&= \frac{\theta\Delta\pi}{r-\mu} + E - \left(\frac{\pi(\alpha)}{r-\mu}\right)\theta_X^{\alpha+1}\theta^{-\alpha}, \qquad (16)
\end{aligned}
$$

where $\Delta\pi \equiv \pi(1) - 2\pi(2) > 0$ and $\pi(\alpha) \equiv \pi(1) + (1-\alpha)\pi(2)/\alpha > \Delta\pi$. $S_M(\theta)$ is a continuously differentiable, increasing and concave function. For $\theta < (>)\theta_X$, $S_M(\theta)$ is less (greater) than zero. The firms bargain over this surplus: a fraction $b \in [0,1]$ is given to the entrant, the remaining fraction $1-b$ to the incumbent; b is treated as a parameter. (Note that the merged firm has no incentive to exit.)

We start by supposing that merger does occur. In section 4.3, we determine the conditions under which this is correct in equilibrium. The entrant's value function V_E (derived, as with all other value functions, in the appendix) is

$$
V_E = \begin{cases} A_E\theta^{\beta} & \text{before entry,} \\ \frac{\theta\pi(2)}{r-\mu} + B_E\theta^{-\alpha} & \text{after entry, before merger,} \\ \frac{\theta\pi(2)}{r-\mu} + B_{E0}\theta^{-\alpha} + bS_M(\theta) & \text{at merger,} \end{cases} \qquad (17)
$$

with the cost $E > 0$ to entry. A_E and B_E are constants determined by boundary conditions. After entry, there is an option term anticipating merger. The value of this option must go to zero as θ becomes large, since merger is not permitted by the policy maker for $\theta > \theta_M$; hence the option value is $B_E\theta^{-\alpha}$, where $\alpha > 0$. The entrant's value function after merger is the sum of its outside option, $\theta\pi(2)/(r-\mu) + B_{E0}\theta^{-\alpha}$ plus its fraction of the surplus from merger, $bS_M(\theta)$.[15] Similarly, the incumbent's value function, V_I, is

$$V_I = \begin{cases} \frac{\theta\pi(1)}{r-\mu} + A_I\theta^{\beta} & \text{before entry,} \\ \frac{\theta\pi(2)}{r-\mu} + B_I\theta^{-\alpha} & \text{after entry, \quad before merger,} \\ \frac{\theta\pi(2)}{r-\mu} + B_{I0}\theta^{-\alpha} + (1-b)S_M(\theta) & \text{at merger.} \end{cases} \tag{18}$$

A_I and B_I are determined by boundary conditions. Finally, the consumer value function, V_C is

$$V_C = \begin{cases} \frac{\theta CS(1)}{r-\mu} + A_C\theta^{\beta} & \text{before entry,} \\ \frac{\theta CS(2)}{r-\mu} + B_C\theta^{-\alpha} & \text{after entry, \quad before merger,} \\ \frac{\theta CS(1)}{r-\mu} & \text{after merger.} \end{cases} \tag{19}$$

With these value functions, strategies take the form of 'threshold rules'.

LEMMA 1: *For any pure strategy chosen by the incumbent, the entrant has a best response of the form "enter immediately if $\theta \in [\theta_E, \infty)$; merge immediately if $\theta \in [0, \theta_M]$". For any pure strategy chosen by the entrant, the incumbent has a best response of the form "merge immediately if $\theta \in [0, \theta_M]$".*

PROOF: See the appendix.

There are two cases to consider. In the first, entry occurs at some $\theta_{E1} > \theta_M$; merger occurs at θ_M, so that the time interval between entry and merger is non-zero. In the second case, entry occurs at $\theta_{E2} \leq \theta_M$ and merger occurs immediately following entry. These two cases are considered separately.

[15]The value after merger can be regarded as the return to the shareholders of the original entrant, who now own a proportion of the merged entity. The same observation applies below to the incumbent.

4.1. Case 1: Delay between Entry and Merger

In this first case, entry occurs at θ_{E1}, chosen optimally by the entrant; both value matching and smooth pasting apply at the entry boundary. There is a constraint on the merger point of the firms (it cannot be greater than θ_M); hence only value matching applies at this point. With this additional condition, the constants in the firms' and consumers' value functions are

$$A_E = \frac{1}{\beta}\left(\frac{\pi(2)}{r-\mu}\right)\theta_{E1}^{-(\beta-1)} - \frac{\alpha}{\beta}B_{E1}\theta_{E1}^{-(\alpha+\beta)} \equiv A_{E1}, \tag{20}$$

$$B_E = B_{E0} + bS_M(\theta_M)\theta_M^{\alpha} \equiv B_{E1}; \tag{21}$$

$$A_I = -\left(\frac{\pi(1)-\pi(2)}{r-\mu}\right)\theta_{E1}^{-(\beta-1)} + B_{I1}\theta_{E1}^{-(\alpha+\beta)} \equiv A_{I1}, \tag{22}$$

$$B_I = B_{I0} + (1-b)S_M(\theta_M)\theta_M^{\alpha} \equiv B_{I1}; \tag{23}$$

$$A_C = \left(\frac{CS(2)-CS(1)}{r-\mu}\right)\left(1-\left(\frac{\theta_M}{\theta_{E1}}\right)^{\alpha+1}\right)\theta_{E1}^{-(\beta-1)} \equiv A_{C1}, \tag{24}$$

$$B_C = -\left(\frac{CS(2)-CS(1)}{r-\mu}\right)\theta_M^{\alpha+1} \equiv B_{C1}. \tag{25}$$

Just as in the previous case, the value matching and smooth pasting conditions give a non-linear equation for the entry trigger θ_{E1}:

$$(\alpha+\beta)B_{E1}\theta_{E1}^{-\alpha} = \beta E - (\beta-1)\frac{\theta_{E1}\pi(2)}{r-\mu}. \tag{26}$$

Note that this equation is very similar to equation (5), with B_{E1} replacing B_{E0} on the left-hand side.

Case 1 holds only in certain circumstances, detailed in the next proposition, The proof and explanation of the proposition are somewhat lengthy and so are relegated to the appendix. The intuition, however, is straightforward. When θ_M is set sufficiently high, the firms will merge as soon as entry occurs. When θ_M is lower, however, there will be (in general) a gap between the merger and entry trigger points. When the state variable has risen to a high enough level, it will be optimal for the entrant to enter as a duopolist; only after θ has fallen are the firms able to merge. Proposition 1 identifies the critical value of θ_M for case 1 to hold.

Proposition 1: *A necessary and sufficient condition for there to exist a solution to equation* (26) *greater than* θ_M *is* $\theta_M \leq \overline{\theta_M}$. *(*$\overline{\theta_M}$ *is defined in the appendix: see definition A.1.)*

Proof: See the appendix.

We now examine the comparative statics of θ_{E1} with respect to various parameters—most importantly, θ_M.

Proposition 2: *The entry trigger* θ_{E1} *(from equation* (26) *when* $\theta_M \leq \overline{\theta_M}$*) is*

- *decreasing in:* $\theta_M, b, \pi(2)$ *and* $\Delta\pi$*;*
- *increasing in:* E.

The proposition establishes the intuitive fact that θ_{E1} is decreasing in θ_M: a more lenient merger policy encourages earlier entry. (The proposition is derived directly from equation (26), using the fact that the term involving θ_M is strictly convex.)

4.2. Case 2: Merger immediately after Entry

In the second case, merger occurs immediately after entry at a level of the state variable $\theta_{E2} \leq \theta_M$. The entry point is chosen optimally by the entrant and so is determined, as in case 1, by value matching and smooth pasting conditions. These conditions give the coefficients

$$A_E = \left(\frac{\alpha+1}{\alpha+\beta}\right)\theta_{E2}^{-(\beta-1)} - \left(\frac{\alpha}{\alpha+\beta}\right)E\theta_{E2}^{-\beta)} \equiv A_{E2}, \tag{27}$$

$$A_I = -\left(\frac{\pi(1)-\pi(2)}{r-\mu}\right)\theta_{E2}^{-(\beta-1)} + B_{I0}\theta_{E2}^{-(\alpha+\beta)} + (1-b)S_M(\theta_{E2})\theta_{E2}^{-\beta} \equiv A_{I2}, \tag{28}$$

$$A_C = 0 \equiv A_{C2}. \tag{29}$$

The post-entry, pre-merger coefficients B_E, B_I and B_C are not stated, since the period over which they apply is negligible. Note that because entry has no effect on market structure—there is monopoly both before and after entry—the consumer value function coefficient A_{C2} equals zero.

From the calculations for case 1 in the appendix, the entry trigger θ_{E2} is given by $\widetilde{\theta_M}$. Two observations follow immediately. First, case 2 applies when $\theta_M \geq \widetilde{\theta_M}$. Secondly, since $\widetilde{\theta_M}$ is not a function of θ_M, merger policy does not affect the entry decision in this case.

4.3. Characterization of Equilibrium

This section considers whether the patterns of entry and merger identified above can be supported in equilibrium. First note that it is always an equilibrium for the incumbent to play "do not merge" and for the entrant to play "enter immediately if θ is in the interval $[\theta_{E0}, \infty)$; after entry, do not merge and exit immediately if θ is in the interval $[0, \theta_X]$". Since merger requires unanimity, these strategies are (weak) best responses to each other.

We now show that for $\theta_M \in [\theta_X, \overline{\theta_M}]$, the strategies

E "enter immediately if θ is in the interval $[\theta_{E1}, \infty)$; after entry, don't exit and merge immediately if θ is in the interval $[0, \theta_M]$, otherwise do not merge";

I "after entry, merge immediately if θ is in the interval $[0, \theta_M]$, otherwise do not merge"

support the equilibrium outcome of entry at $\theta_{E1} > \theta_M$, merger at θ_M. Merger is allowed only when $\theta \leq \theta_M$. With this in mind, we abbreviate the incumbent's strategy to "merge at θ_M", and the entrant's strategy to "enter at θ_{E1}, merge at θ_M".

From the construction of the value functions, only deviations in the continuation game after entry (that is, only decisions concerning merger) need to be considered. To see this, notice that

$$A_{Ei} = \left(\frac{\alpha+1}{\alpha+\beta}\right)\left(\frac{\theta_{Ei}\pi(2)}{r-\mu} - \left(\frac{\alpha}{\alpha+1}\right)E\right)\theta_{Ei}^{-\beta}$$

is decreasing (increasing) in θ_{Ei} iff θ_{Ei} is greater (less) than

$$\left(\frac{\alpha}{\alpha+1}\right)\left(\frac{\beta}{\beta-1}\right)\left(\frac{E}{\pi(2)}\right)(r-\mu),$$

for $i \in \{0,1\}$. Both θ_{E0} and θ_{E1} are greater than this critical value. The argument below establishes that $\theta_M \geq \theta_X$; hence $B_{E1} \geq B_{E0}$, which in turn implies that $\theta_{E1} \leq \theta_{E0}$: see equations (5) and (26). Therefore $A_{E1} \geq A_{E0}$, and deviations in the continuation games before entry (conditional on merger occurring at θ_M after entry) are not profitable.

Suppose that the incumbent plays "merge at θ_M", and consider the continuation game after entry at θ_{E1}. There are two types of deviation for the entrant: (i) those that involve merger at some lower level of the state variable; (ii) those that do not involve merger, and so involve exit at θ_X. It is straightforward to show, using the arguments in the proof of lemma 1, that the first deviation yields a lower expected payoff to the entrant. The second deviation yields a lower expected payoff to the entrant iff $B_{E1} \geq B_{E0}$ i.e., iff $S_M(\theta_M) \geq 0$ (from equations (8) and (21)). This inequality is equivalent to $\theta_M \geq \theta_X$.

Suppose that the entrant plays "enter at θ_{E1}, merge at θ_M". There are two types of deviation for the incumbent: (i) those that involve merger at some lower level of the state variable; (ii) those that do not involve merger. Consider the deviation "merger at $\theta_{\mathcal{M}} < \theta_M$". The change in the incumbent's value function after entry but before merger from a marginal change in the merger point is positive (from equations (18) and (23)). The incumbent will therefore play either "merge at θ_M" or "do not merge", according to which yields the higher expected payoff. From equations (9) and (18), the incumbent prefers "merge at θ_M" iff $B_{I1} \geq B_{I0}$; from equation (23), this inequality is equivalent to $\theta_M \geq \theta_X$.

$\overline{\theta_M}$ is determined by the non-linear equations (A7) and (A9), and so it is not possible to give analytical conditions under which $\theta_X \leq \overline{\theta_M}$. It can be demonstrated that there are parameter values under which this inequality holds. Note that $\overline{\theta_M} = \widehat{\theta_M}$ if b is sufficiently large.[16] Assumption 1 then ensures that $\widehat{\theta_M} > \theta_X$. The numerical analysis in section 6 provides further demonstration that there are parameter values such that $\overline{\theta_M} \geq \theta_X$.

[16] $\overline{\theta_M} = \widehat{\theta_M}$ if $\widehat{\theta_{E1}} > \widehat{\theta_M}$. A sufficient condition for the latter inequality is $b > \max[\beta/((\alpha+1)(\alpha+\beta)), 1/(1+\alpha(\pi(1)/\pi(2)-1))]$. Note that the lower bound on b is less than 1.

The preceding discussion can be summarized:

PROPOSITION 3: *The 'no-merger' strategies*

- E *"enter immediately if* θ *is in the interval* $[\theta_{E0}, \infty)$*; after entry, do not merge and exit immediately if* θ *is in the interval* $[0, \theta_X]$*";*

- I *"do not merge"*

are a Markov Perfect equilibrium for all values of θ_M*.*

If $\theta_M \in [\theta_X, \overline{\theta_M}]$*, then there is a second Markov Perfect ('merger') equilibrium in which the firms' strategies are:*

- E *"enter immediately if* θ *is in the interval* $[\theta_{E1}, \infty)$*; after entry, don't exit and merge immediately if* θ *is in the interval* $[0, \theta_M]$*, otherwise do not merge";*

- I *"after entry, merge immediately if* θ *is in the interval* $[0, \theta_M]$*, otherwise do not merge".*

If $\theta_M > \overline{\theta_M}$*, then the second Markov Perfect equilibrium is:*

- E *"enter immediately if* θ *is in the interval* $[\theta_{E2}, \infty)$*; after entry, don't exit and merge immediately";*

- I *"after entry, merge immediately.*

Hence when $\theta_M \in [\theta_X, \overline{\theta_M}]$, there are two equilibria of the game. The derivation of the merger equilibrium shows, however, that when it exists, it is preferred by both firms to the no-merger equilibrium. In this case, it is natural to select this equilibrium. Finally, for completeness, we note that when $\theta_M \geq \overline{\theta_M}$, there is an equilibrium in which entry occurs at $\widetilde{\theta_M}$ and merger occurs immediately after entry. Since there is no change in market structure in this case (there is monopoly both before and after entry), it is clear that such a merger policy is unlikely to be chosen by a social planner. This leads us on to the analysis of optimal merger policy.

Consider now the planner's choice of θ_M in order to maximize social surplus. Assume that the planner commits to the choice of θ_M at the outset i.e., before entry. θ_M is chosen to maximize the social value function—the consumers' value function plus the sum of the firms' value functions, weighted by $\lambda \in [0,1]$. When $\theta_M \in [\theta_X, \overline{\theta_M}]$, we assume that the (Pareto preferred) merger equilibrium is selected; the social value function is then

$$\begin{aligned} V_{M1}^S(\theta_M) &= V_{C1} + \lambda(V_{I1} + V_{E1}), & (30)\\ &= \frac{\theta(CS(1) + \lambda\pi(1))}{r-\mu} + (A_{C1}(\theta_M) + \lambda(A_{I1}(\theta_M) + A_{E1}(\theta_M)))\,\theta^\beta, & (31)\end{aligned}$$

where A_{E1}, A_{I1} and A_{C1} are as determined by equations (20), (22) and (24). The value function has been written to emphasize that it is a function of θ_M, through the coefficients A_{E1}, A_{I1} and A_{C1}.

When $\theta_M < \theta_X$, the no-merger equilibrium is selected and the social value function is

$$V_{NM}^S = \frac{\theta(CS(1) + \lambda\pi(1))}{r-\mu} + (A_{C0} + \lambda(A_{I0} + A_{E0}))\,\theta^\beta, \tag{32}$$

where A_{E0}, A_{I0} and A_{C0} are as determined by equations (7), (10) and (13). When $\theta_M > \overline{\theta_M}$, either no merger occurs, or merger occurs immediately upon entry. In the latter case, the social value function is

$$V_{M2}^S = \frac{\theta(CS(1) + \lambda\pi(1))}{r-\mu} + (A_{C2} + \lambda(A_{I2} + A_{E2}))\,\theta^\beta, \tag{33}$$

where A_{E2}, A_{I2} and A_{C2} are as determined by equations (27), (28) and (29). In both of these cases, the value functions V_{NM}^S and V_{M2}^S do not depend on θ_M.

For tractability, the analysis in this section concentrates on the case of $\lambda = 0$ i.e., the social planner cares only about consumer surplus. Note that in this case, the social planner will not set θ_M so that case 2 occurs, since $A_{C2} = 0$, and $A_{C0} \geq 0$. Hence the relevant choice for the planner is whether to disallow merger (or equivalently, set $\theta_M \leq \theta_X$ to ensure the no-merger outcome); or to set $\theta_M \in (\theta_X, \overline{\theta_M}]$ and face the merger equilibrium. A necessary and sufficient condition for social welfare to be higher when merger is allowed is $A_{C1} \geq A_{C0}$; that is,

$$\left(1 - \left(\frac{\theta_M}{\theta_{E1}}\right)^{\alpha+1}\right) \theta_{E1}^{-(\beta-1)} \geq \left(1 - \left(\frac{\theta_X}{\theta_{E0}}\right)^{\alpha+1}\right) \theta_{E0}^{-(\beta-1)}. \tag{34}$$

When $\theta_M = \theta_X$, $\theta_{E1} = \theta_{E0}$ (see equations (5) and (26) defining the entry triggers). Hence $A_{C1} = A_{C0}$ when $\theta_M = \theta_X$. If A_{C1} is strictly increasing in θ_M at $\theta_M = \theta_X$, then there is a range of values for θ_M over which $A_{C1} > A_{C0}$. The next proposition deals with this case.

Proposition 4: *There exists a $\theta_M^C > \theta_X$ such that merger policy increases consumer surplus for $\theta_M \in (\theta_X, \theta_M^C]$ if and only if both of the following conditions are satisfied:*

$$\left(\frac{\beta-1}{\alpha+\beta}\right)\left(\frac{\theta_{E0}}{\theta_X}\right)^{\alpha+1} > 1, \tag{35}$$

$$(\alpha+\beta)b\left(\frac{\Delta\pi}{\pi(2)} + 1\right) > 1. \tag{36}$$

Proof: See the appendix.

Proposition 4 gives a set of conditions under which it is optimal for the policy-maker to set the merger trigger θ_M above θ_X. The next proposition interprets the conditions in terms of the model parameters.

Proposition 5:

- *There exists a $b^* \in [0,1]$ such that the necessary and sufficient conditions in proposition 4 are satisfied when $b \geq b^*$.*
- *If $b > 0$, then there exists a $\Pi^* \geq 0$ such that the necessary and sufficient conditions in proposition 4 are satisfied when $\Delta\pi/\pi(2) \geq \Pi^*$.*
- *If $\mu = 0$ and $b > 0$, then there exists a $\sigma^* \leq +\infty$ such that the necessary and sufficient conditions in proposition 4 are satisfied when $\sigma \leq \sigma^*$.*

Proof: See the appendix.

Other things equal, the consumerist social planner prefers a higher merger trigger ($\theta_M > \theta_X$) if increasing θ_M induces a large enough fall in the entry trigger θ_{E1}. The response of θ_{E1} to an increase in θ_M is determined by the effect of the increase on the share of the merger surplus, $bS_M(\cdot)$, received by the entrant. The entrant's marginal surplus, evaluated at $\theta_M = \theta_X$, is:

$$b \left.\frac{\partial S_M(\theta_M)}{\partial \theta_M}\right|_{\theta_M=\theta_X} = b\left(\frac{\Delta\pi + \alpha\pi(\alpha)}{r-\mu}\right).$$

$S_M(\cdot)$ is an increasing function, and so this marginal surplus (evaluated at $\theta_M = \theta_X$) is positive. The greater the marginal surplus, the greater the decrease in θ_{E1} for a small increase in θ_M from θ_X.

The first two conditions in proposition 5 are then easy to understand. When the entrant's share of the merger surplus, b, is large, then so is the entrant's marginal surplus. Consequently, an increase in θ_M produces a large decrease in θ_{E1}. In contrast, when $b = 0$, so that the entrant receives none of the surplus, its entry point is θ_{E0}, whatever the value of θ_M. Similarly, when there is little relative gain from merger—that is, $\Delta\pi/\pi(2)$ is small—merger policy has little effect on the entry decision. For larger merger gains, however, the entry decision is more responsive.

The least obvious condition relates to uncertainty. Due to the non-linearity of the expressions, we are able to obtain an analytical result only when $\mu = 0$; by continuity, however, the conclusion holds for a positive μ that is not too large. Differentiation of the entrant's marginal surplus shows that

$$\frac{\partial}{\partial \sigma}\left(\left.\frac{\partial S_M(\theta_M)}{\partial \theta_M}\right|_{\theta_M=\theta_X}\right) = \left(\frac{\pi(1)-\pi(2)}{r-\mu}\right)\frac{\partial \alpha}{\partial \sigma} < 0.$$

Hence the entrant's marginal surplus at $\theta_M = \theta_X$ is greatest when σ is small, and declines as σ increases. Consequently, θ_{E1} decreases most quickly (for a small increase in θ_M above θ_X) when uncertainty is low. The reason is that higher uncertainty increases the marginal value of the firms' outside options (relating to exit). The value of the options are $(\pi(\alpha)/(r-\mu))\,\theta_X^{\alpha+1}\theta_M^{-\alpha}$; hence the marginal value at $\theta_M = \theta_X$ is $-\alpha\pi(\alpha)/(r-\mu)$. This marginal value is increasing in σ. Hence, the greater the degree of uncertainty, the more

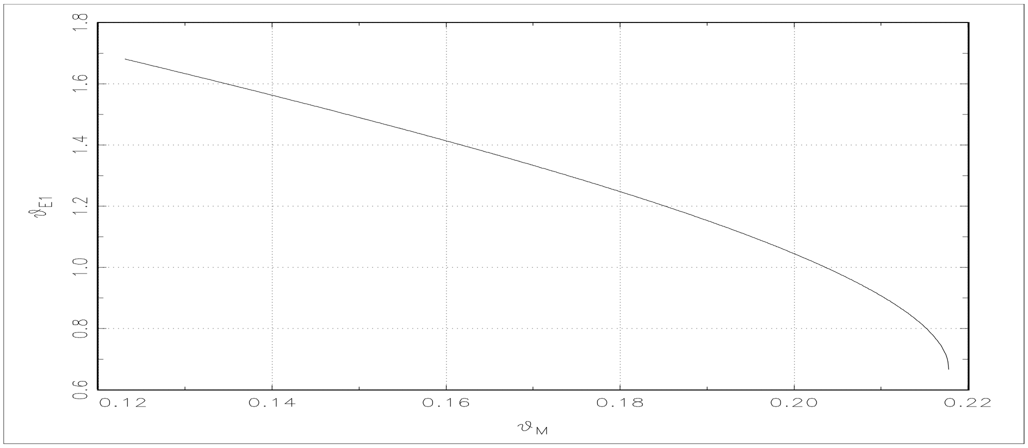

Figure 1: θ_{E1} as a function of θ_M

valuable is the option to exit, both in level and at the margin. In summary, then, when σ is large, the (relative) value of merger to the entrant is low, θ_{E1} decreases slowly with θ_M, and a lenient merger policy is less likely to be socially optimal.

6. Numerical Analysis

In this section, we illustrate the model and results using a numerical example. With the parameter values used,[17] the exit trigger $\theta_X = 0.1231$, the entry trigger in the no-merger equilibrium $\theta_{E0} = 1.6812$, and the critical value of the merger trigger point $\overline{\theta_M} = 0.2178$.

Figure 1 shows the dependence of θ_{E1} on θ_M as θ_M varies from θ_X to $\overline{\theta_M}$. It is, as shown in proposition 2, a decreasing function, ranging from 1.6812 to 0.6669. Figure 2 shows the dependence of θ_{E1} on three other parameters: b, σ and r.[18] The comparative static with respect to b is as predicted by proposition 2: as the share of the merger surplus to the entrant increases, the entry trigger decreases. The comparative static with respect to σ (which is not considered in the proposition) is more complicated. At low levels of uncertainty, the

[17] The values used are: $\mu = 0, \sigma = 0.6, r = 0.05, E = 40, \pi(1) = 10, \pi(2) = 3, CS(1) = 2, CS(2) = 8$ and $b = 0.5$.

[18] In all cases, θ_M has been set equal to 0.16.

entry trigger increases with σ. At higher levels of σ, however (above 0.43 in this example), the entry trigger decreases with σ. There are two effects. The first is standard in the real options literature: increased uncertainty induces delay in irreversible investment (i.e., entry) for any given payoff from investment; see Dixit and Pindyck (1994) The second is that the payoff from entry increases because the value of the entrant's outside option increases. This second effect decreases the entry trigger. In this example, the first effect dominates when σ is small, the second when σ is large. Finally, the figure indicates that θ_{E1} is increasing in r (a comparative static again not considered in the proposition). This is the standard comparative static for entry or investment triggers in a real options setting—an increase in the interest rate raises the value of the option to enter, increasing the opportunity cost of entering now, and so delaying entry. Again, the intuition is complicated, however, by the presence of the outside option of exit.

Figure 3 shows the social value function coefficients A_{C0} and A_{C1} when $\lambda = 0$ for different values of θ_M. With these parameter values, $(\theta_X/\theta_{E0})^{\alpha+1} = 0.0405$ and $(\beta - 1)/(\alpha + \beta) = 0.1559$; and $(\alpha+\beta)b\left(\frac{\Delta\pi}{\pi(2)} + 1\right) = 1.6951$. Hence the conditions in proposition 4 are satisfied, and we expect A_{C1} to be increasing in θ_M when $\theta_M = \theta_X$. The figure confirms that this is indeed the case. In fact, in this example, there is an interior optimal value of θ_M, equal to 0.1819.

The analytical results are confined to the consumerist case when $\lambda = 0$. Numerical analysis can investigate how the optimal merger point changes for larger values of λ. With these parameter values, the sum of the firms' value function coefficients $A_{E1} + A_{I1}$ is decreasing in θ_M: see figure 4. In this example, therefore, we expect θ_M^* to be decreasing in λ. This is confirmed in figure 5, which shows that θ_M^* decreases from 0.1819 when $\lambda = 0$ to 0.1668 when $\lambda = 0.1460$. For values of λ above 0.1460, the social planner prefers the no-merger equilibrium.

Finally, we can investigate numerically how the optimal merger point depends on other parameters in the model. Figure 6 indicates that θ_M^* is increasing in b, decreasing in σ and increasing in r.[19]

[19] The lines in the figure are not smooth because a relatively coarse grid of parameter values was used in the numerical routines, to save on computation time.

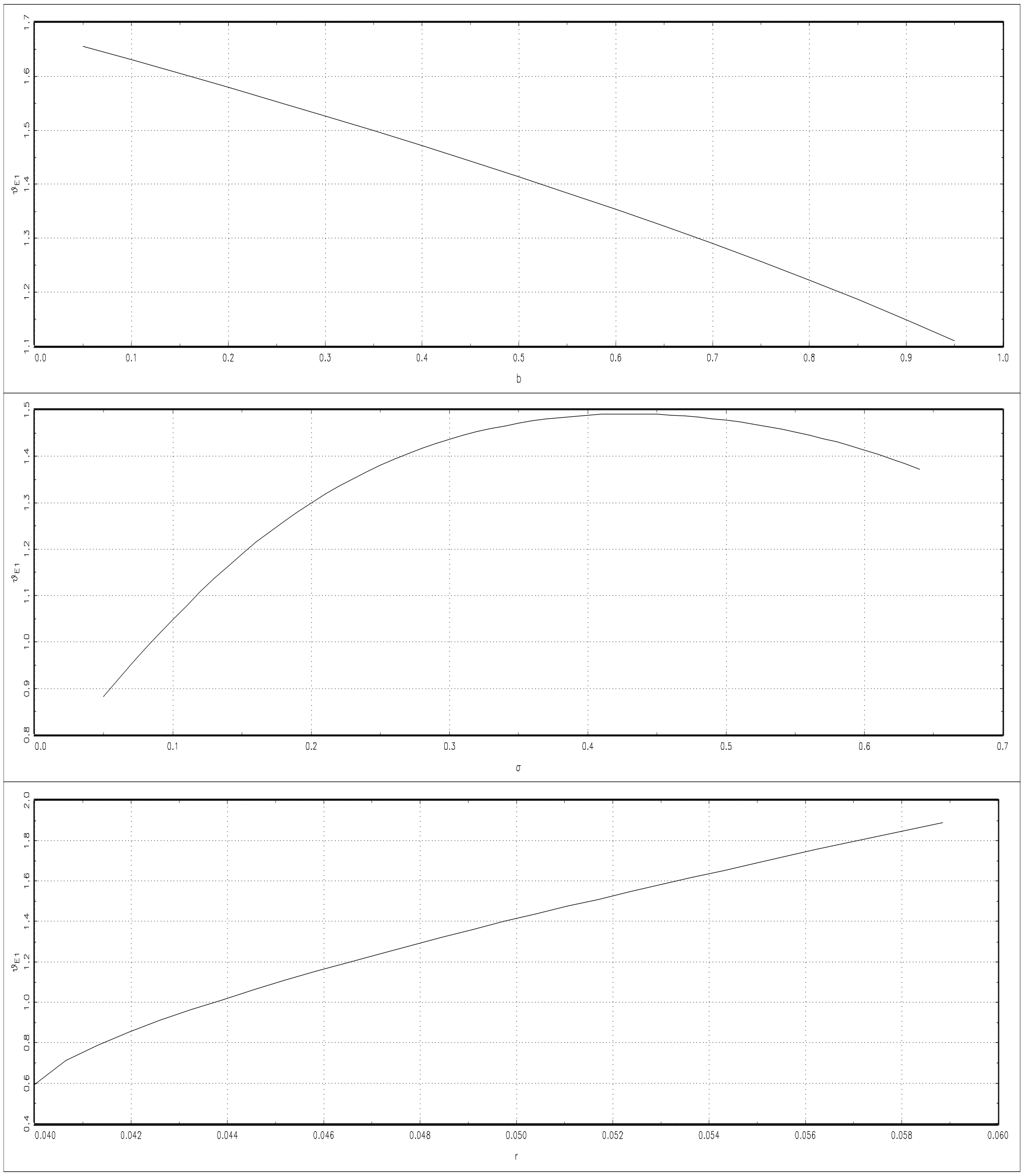

Figure 2: The comparative statics of θ_{E1}

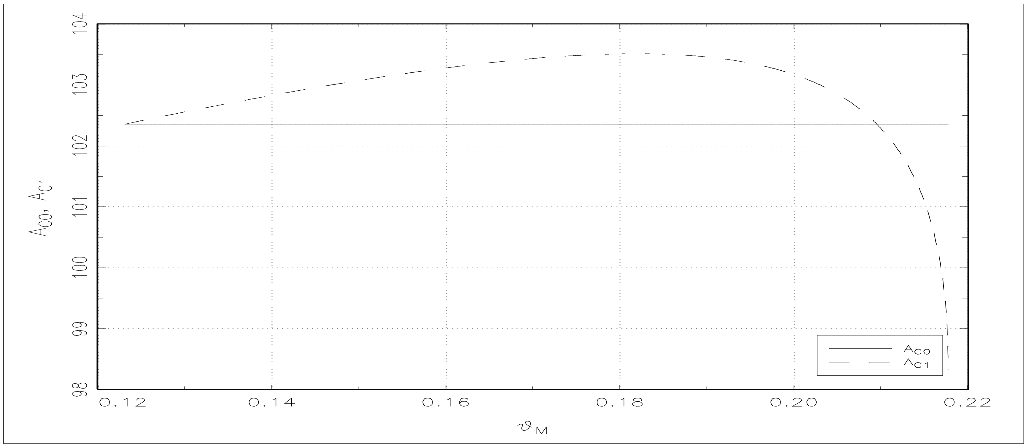

Figure 3: The social value function coefficients when $\lambda = 0$

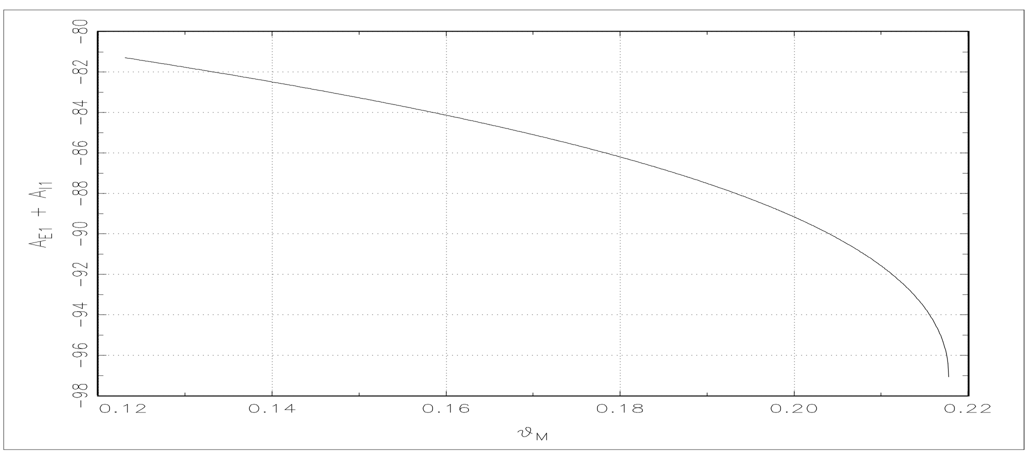

Figure 4: The sum of the firms' value function coefficients

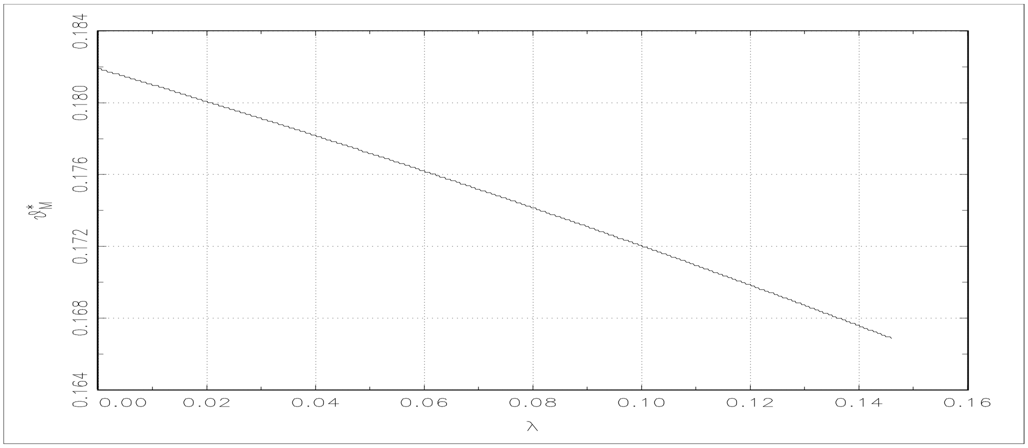

Figure 5: θ^*_M as a function of λ

The optimal merger point is determined by the balance between the marginal benefit from hastening entry (decreasing θ_{E1}) and the marginal cost of increasing concentration at times of financial distress (increasing θ_M, holding θ_{E1} constant). As b increases, θ_{E1} decreases; see proposition 2 and figure 2. More importantly for the comparative static, the entrant's marginal surplus from merger increases with b (see the discussion in the previous section). As a result, θ_{E1} is more responsive to changes to b when b is large; this can be seen in figure 2, which shows that θ_{E1} is concave in b. Hence the marginal benefit from hastening entry is greater when b is higher; as a result, the optimal merger point is higher.

The second comparative static, with respect to σ, is more complicated, for two reasons. First, θ_{E1} is a non-monotonic function of the degree of uncertainty σ (see figure 2). When σ is sufficiently large, θ_{E1} is a decreasing function of σ; it might then be expected that an increase in σ would decrease the optimal merger point. When σ is sufficiently small, θ_{E1} is an increasing function of σ; it might then be expected that an increase in σ would increase the optimal merger point. Secondly, a change in σ affects both sides of the marginal equality determining the optimal merger point. Increased uncertainty, holding θ_{E1} constant, increases both the marginal benefit and cost associated with merger policy. The outcome is, therefore, ambiguous *a priori*. Figure 6 shows how these various factors resolve with these parameter values; it suggests that the optimal merger point θ^*_M decreases with σ.

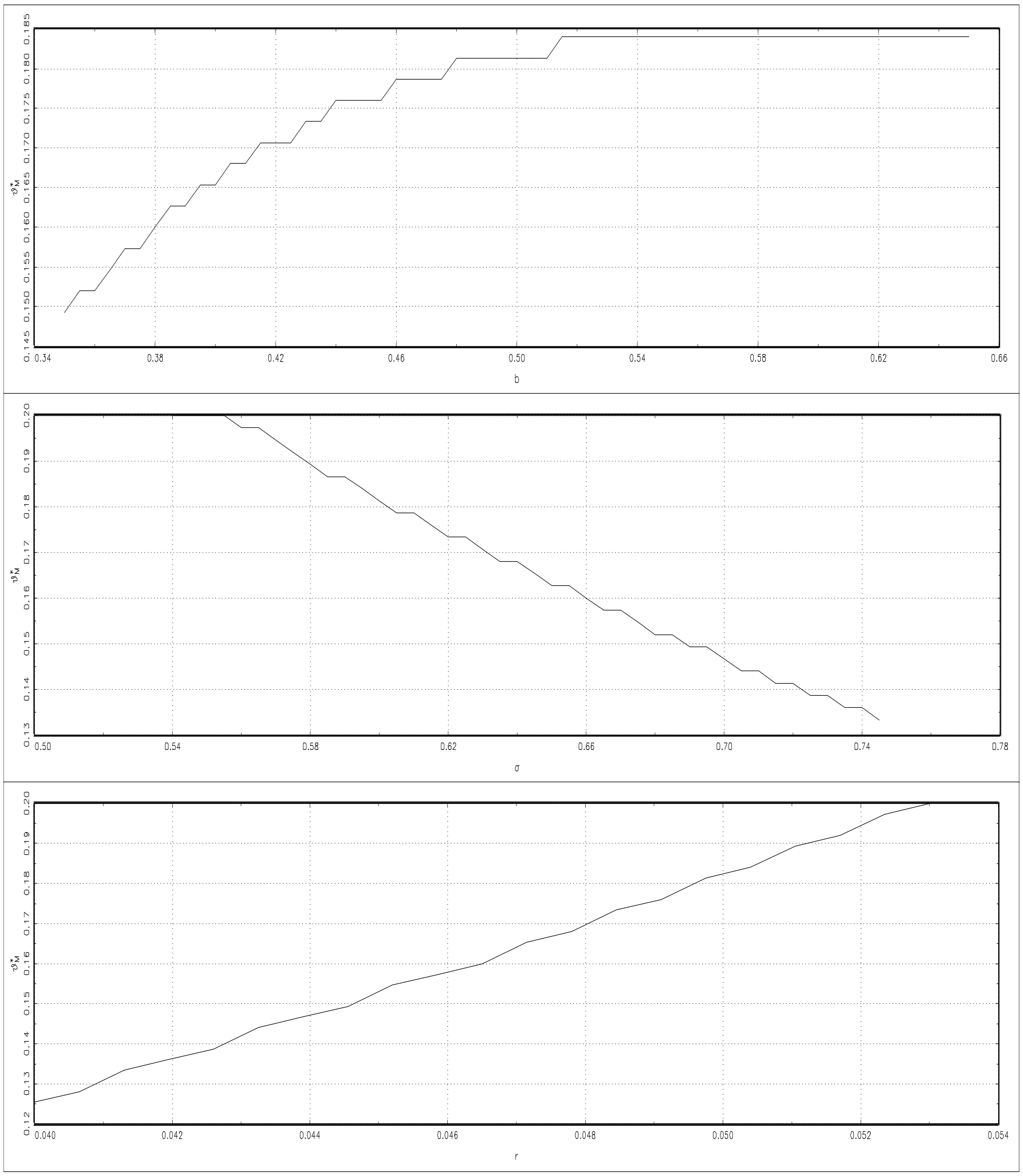

Figure 6: The comparative statics of the optimal merger point

Finally, the figure indicates that the optimal merger point is increasing in r. Earlier numerical analysis of θ_{E1} shows that it is increasing in r; see figure 2. When r is high, therefore, so is θ_{E1} and, other things equal, entry occurs relatively late. The planner's optimal response to this is to increase θ_M: this encourages earlier entry (decreases θ_{E1}); the discounted cost of earlier merger is relatively low.

7. Discussion

We have found that more lenient merger policy towards failing firms may be beneficial to consumers and increase social welfare. Despite the fact that consumer surplus is decreased when merger is permitted, the increase in consumer surplus resulting from earlier entry may more than offset this loss. In fact, it is the consumerist social planner that adopts the most lenient merger policy, as this planner takes no account of the overall reduction in industry profits. This argument, while phrased in terms of entry, applies to any *ex ante* investment decision that increases social surplus.

Some intuition can be offered for this somewhat surprising result. The gain in consumer surplus from earlier entry outweighs the loss from earlier merger for two reasons. First, merger occurs later than entry; hence its welfare impact is deferred and subject to discounting, while the gain from entry is immediate. Secondly, merger occurs at a time when, due to the low value of θ, all values—the profits of both firms and consumer surplus—are low. By contrast, entry occurs when θ—and hence the consumer surplus gain from entry—is high.

In effect, strict merger control provides the incumbent with highly desirable commitment power, similar to Rasmusen (1988). Prior to entry, the incumbent would like to threaten never to merge with the entrant. In the absence of any means of committing to this strategy, however, this threat is not credible—following the fact of entry, merger benefits the incumbent more than continued duopoly. Thus, entry followed by merger (when permitted) is a subgame perfect equilibrium. By preventing anti-competitive mergers, even in situations of financial distress, strict merger control provides the incumbent with the commitment power it need to prevent, or at least delay, entry. A more lenient merger policy weakens this commitment, reducing its deterrent effect.

A related intuition is that a more lenient merger policy reduces the sunkness of entry. It would therefore be expected to result in higher levels of both entry and exit (merger). Given that the decisions are related, the policymaker faces a trade-off between the two. This paper provides a framework for finding the optimum in this situation, which in many instances would seem to involve a more lenient approach than currently seen in U.S. and E.U. merger control.

In the model presented here, certain assumptions concerning market structure have been made for analytical tractability. Of course, other assumptions are also made, but this one raises interesting issues for future research. In our model, one firm (the incumbent) always operates in the market; the other firm decides when/whether to enter and exit. This imposed asymmetry simplifies the analysis considerably to concentrate on the main idea: that merger increases post-entry expected profits, and so hastens entry. (We observe in passing that this may not be that bad a description of markets in which entry occurs. Geroski (1995) notes that most entry is *de novo* i.e., by non-incumbent firms; and that most exit is by young, new firms.) An alternative market structure would have *ex ante* symmetric firms with endogenous determination of the order of entry and exit. In this formulation, one firm enters first as the 'leader'—thus becoming the 'incumbent' in our model—while the other enters strictly later as the 'follower' or 'entrant'.[20] Due to the incentive to pre-empt its rival, the entry point of the leader is, in a duopoly, determined by rent equalization—the point at which the value function of the leader equals the value function of the follower; see Fudenberg and Tirole (1985).

We have demonstrated in this paper that the impact of more lenient merger policy is to hasten the follower's entry. The effect on the leader, however, is ambiguous, depending on the relative effect on the value functions of the leader and follower. Two polar cases can readily be considered. If $b = 1$, then the leader receives no surplus from the merger and an increase in the permitted merger point θ_M raises the follower's value function while leaving that of the leader unchanged. As a result, the incentive to preempt is weaker and the leader will enter the industry later. If $b = 0$, on the other hand, the follower's value function is unaffected while the leader's value function increases; the leader enters earlier while the

[20]For certain parameter values simultaneous entry is also possible: for details see Weeds (2002). The discussion here focusses on the leader-follower equilibrium where firms enter sequentially.

follower's behaviour is unaffected. By continuity, there exists a critical value b^* such that more lenient policy hastens the leader's entry for $b < b^*$ and delays it for $b > b^*$. Thus, an increase in θ_M has an ambiguous effect on the time to first entry—depending on the size of b—but decreases the time to second entry. The overall welfare effect is therefore somewhat more complicated than in the market structure that has been used here.

In the model presented in this paper, the mode of competition between the firms is taken as given and does not change with the policy rule or the level of θ. In other words, the profit levels $\pi(1)$ and $\pi(2)$ are fixed parameters: $\pi(1)$ is the monopoly profit, while $\pi(2)$ might be taken to be the duopoly profit level in a symmetric Cournot model. However, it is possible that the firms—particularly the incumbent—may wish to deviate from short-run profit maximisation to hasten exit or merger. In short, the possibility of predation may alter the analysis.

Predatory actions which reduce the profitability of the entrant will hasten the time at which the merger threshold is reached (though it should be noted that, as in the standard analyses, consumer surplus is likely to be higher during the intervening period). It is unclear how the incentive for predation changes with the level of θ—i.e., does predation become more or less attractive as industry profitability falls?—or how this is affected by a more lenient merger policy. Greater leniency makes the object of predation (the removal of the rival) easier to achieve; but it also makes predatory actions less necessary as a means of inducing exit. It is unclear what would be the overall implication of greater leniency for predation. A framework in which firms have some discretion over their pricing as well as entry, exit and merger decisions would allow this issue to be investigated; see Saloner (1987) for an early analysis.

We have modelled uncertainty as affecting both firms in the industry; that is, we have modelled a failing industry. We could equally well have used a model in which uncertainty affects only the entrant and not the incumbent i.e., have modelled one failing firm. This may have quantitative effects on the welfare analysis. In the version we have analysed, exit occurs when the state variable is at a low level; this means that the deadweight loss from allowing merger is relatively low. In the version where uncertainty affects only the entrant, merger could occur in a state where the deadweight loss from merger is relatively high. Overall, however, there would be little qualitative difference to our conclusions.

Finally, the analysis in the paper assumes that the competition authority can commit to the merger control rule. It is important for the analysis that the rule is not altered after entry has occurred, despite the desirability of this action regarding the particular industry at the time of failure. This emphasises the importance of establishing the rule as a clear and general policy and implementing it consistently thereafter. Time-consistency on the part of the regulator may be ensured through the repeated nature of the interactions across industrial sectors: a regulator that breaches the policy rule in one instance loses its credibility in all industries.

8. Conclusions

We have argued that assessment of the failing firm defence in merger cases should take into account the effect of the policy rule on the incentives for entry (and *ex ante* investment decisions in general). A more lenient policy—which could be characterised as permitting the defence to be used by 'flailing' as well as imminently failing firms—may yield social benefits through its beneficial impact on entry, resulting in more effective competition in the long run. This paper provides a framework for determining the optimal degree of leniency, which balances the losses from increasing concentration after merger with the gains from hastening entry and competition.

This view challenges several of the conclusions that have been reached by policymakers. In particular, three assumptions underlying policy and/or practice in this area are questioned. The first is that a consumerist social planner (e.g., a competition authority) should be the most strict in implementing merger control. By contrast, in this model it is the consumerist authority that adopts the most lenient merger rule.

Secondly, the share of the merger surplus granted to the failing firm is important, but in a way that differs from the view adopted by competition authorities in certain cases. In the Detroit newspaper JOA, the (equal) share given to the 'failing firm' cast doubt on the relevance of the failing firm defence to this case. By contrast, this paper argues that the beneficial effect of a more permissive merger policy on entry is reduced if the share given to the failing firm is small. If the failing firm defence is less likely to be accepted in cases where the share given to the failing firm is reasonably significant, the wider benefits of the policy will not be realised.

Thirdly, a failing firm that has greater bargaining power, perhaps because it is a division of a large corporate group, gains a greater share of the surplus from merger and its entry decision will be more sensitive to the merger rule. Thus, the outcome of the *ICI-Kemira Oy* case, in which a failing division was judged more harshly than may have been the case for a stand-alone firm in a similar financial position, also threatens to undermine the benefit of the policy. The failing firm defence can generate greater welfare gains in cases where the target gains a substantial share of the surplus than in those where the failing firm has little bargaining power.

APPENDIX

A.1. Derivation of Value Functions

This section will derive the entrant's value function for the case when merger is not permitted. The derivations of all other value functions are very similar and so are omitted.

The entrant's value function $V_E(\theta_t)$ at time t before it has entered, given a level θ_t of the state variable, is

$$V_E(\theta_t) = \sup_{T_E, T_X} \mathbb{E}_t \Big[- E \exp\left(-r(T_E - t)\right) + \int_{T_E}^{T_X} \exp\left(-r(\tau - t)\right)\theta_\tau \pi(2) d\tau + E \exp\left(-r(T_X - t)\right) \Big] \quad \text{(A1)}$$

where T_E and T_X are the random times of entry and exit, the operator $\mathbb{E}_t$ denotes expectations conditional on information available at time t, and the sup is taken over all entry and exit ('stopping') times T_E and T_X with the process $\{\theta_t\}$.

In this 'continuation' region, in any short time interval dt starting at time t the entrant receives a flow payoff of 0 and experiences a capital gain or loss dV_E. The Bellman equation for the value of the entry opportunity is therefore

$$V_E = \exp\left(-r dt\right)\mathbb{E}_t \left[V_E + dV_E\right]. \quad \text{(A2)}$$

Itô's lemma and the GBM equation (1) gives the ordinary differential equation (ODE)

$$\frac{1}{2}\sigma^2\theta^2 V_E''(\theta) + \mu\theta V_E'(\theta) - rV_E(\theta) = 0. \quad \text{(A3)}$$

From equation (1), it can be seen that if θ ever goes to zero, then it stays there forever. Therefore the option to enter has no value when $\theta = 0$, and must satisfy the boundary condition $V_E = 0$. Solution of the differential equation subject to this boundary condition gives $V_E = A_E\theta^\beta$, where A_E is a positive constant and $\beta > 1$ is the positive root of the quadratic equation $\mathcal{Q}(z) = \frac{1}{2}\sigma^2 z(z-1) + \mu z - r$.

Now consider the value of the entrant in the region in which the value of θ is such that it is optimal to enter at once. The entrant's value function is then

$$V_E(\theta_t) = \sup_{T_X} \mathbb{E}_t \left[\int_t^{T_X} \exp\left(-r(\tau - t)\right) \left(\theta_\tau \pi(2)\right) d\tau + E \exp\left(-r(T_X - t)\right) \right]. \tag{A4}$$

In any short time interval dt starting at time t, the entrant receives a flow payoff of $\theta\pi(2)$, plus a capital gain or loss dV_E relating to the possibility of exit. The Bellman equation for the entrant's value after entry but before exit is therefore

$$V_E = \theta\pi(2) + \exp\left(-rdt\right)\mathbb{E}_t \left[V_E + dV_E\right]. \tag{A5}$$

Itô's lemma and the GBM equation (1) gives the ODE

$$\frac{1}{2}\sigma^2\theta^2 V_E{}''(\theta) + \mu\theta V_E{}'(\theta) - rV_E(\theta) + \theta\pi(2) = 0. \tag{A6}$$

As θ becomes arbitrarily large, the component of the value function relating to exit should become arbitrarily small. Solution of the differential equation with this boundary condition gives $V_E = \theta\pi(2)/(r - \mu) + B_E\theta^{-\alpha}$, where B_E is a negative constant and $\alpha > 0$ is the positive root of the quadratic equation $\mathcal{Q}(z) = \frac{1}{2}\sigma^2 z(z+1) - \mu z - r$.

A.2. Proofs

Proof of Lemma 1

Suppose that the incumbent plays a strategy "merge if $\theta \in \Theta_I \subseteq \mathbb{R}_+$". Consider the continuation game after entry, and a strategy by the entrant of "merge if $\theta \in \Theta_E \subseteq \mathbb{R}_+$". Value matching holds at any $\theta_{\mathcal{M}} \leq \theta_M$ at which merger occurs, since value functions are forward-looking; hence $B_E\theta_{\mathcal{M}}^{-\alpha} = B_{E0}\theta_{\mathcal{M}}^{-\alpha} + bS_M(\theta_{\mathcal{M}})$. Both $B_E\theta^{-\alpha}$ and $B_{E0}\theta^{-\alpha} + bS_M(\theta)$ are continuously differentiable, decreasing functions of θ. At $\theta_{\mathcal{M}}$, the derivative of $B_E\theta^{-\alpha}$ is greater than the derivative of $B_{E0}\theta^{-\alpha} + bS_M(\theta)$. Hence $B_E\theta^{-\alpha}$ is greater (less) than $B_{E0}\theta^{-\alpha} + bS_M(\theta)$ as θ is less (greater) than $\theta_{\mathcal{M}}$. Hence the entrant's best response is "merge immediately if $\theta \in [0, \theta_M]$". An identical argument establishes an equivalent result for the incumbent's best response. The form of the entrant's strategy with respect to entry follows from standard calculations.

Proof of Proposition 1

Two potential issues arise for the solution to equation (26). First, a solution to this equation may not exist. Secondly, the relevant solution to equation (26) may be less than θ_M, which would contradict the requirement of case 1 that $\theta_{E1} \geq \theta_M$. Proposition 1 establishes an upper bound $\overline{\theta_M}$ on θ_M such that a solution to equation (26) that is greater than θ_M exists iff $\theta_M \leq \overline{\theta_M}$. To prove the proposition, we use the following lemma.

LEMMA A.1: *Assuming that $\theta_{E1} \geq \theta_M$, a necessary and sufficient condition for there to exist at least one solution to equation* (26) *is $\theta_M \leq \widehat{\theta_M}$, where $\widehat{\theta_M}$ is given by the non-linear equation*

$$S_M(\widehat{\theta_M})\widehat{\theta_M}^{\alpha} = \frac{1}{b}\left[\left(\frac{\beta E}{(\alpha+1)(\alpha+\beta)}\right)\widehat{\theta_{E1}}^{\alpha} - \frac{1}{\alpha}\left(\frac{\pi(2)}{r-\mu}\right)\theta_X^{\alpha+1}\right], \tag{A7}$$

$$\text{where} \quad \widehat{\theta_{E1}} \equiv \left(\frac{\alpha}{\alpha+1}\right)\left(\frac{\beta}{\beta-1}\right)\left(\frac{E}{\pi(2)}\right)(r-\mu). \tag{A8}$$

PROOF: The left-hand side of equation (26) is strictly positive and convex in θ_{E1}; the right-hand side is linear in θ_{E1}. Define $\widehat{\theta_{E1}}$ to be the value of θ_{E1} where the slopes of the convex and linear components are equal: $\alpha(\alpha+\beta)B_{E1}\widehat{\theta_{E1}}^{-(\alpha+1)} = (\beta-1)\pi(2)/(r-\mu)$. Hence there are no/one/two solutions to the equation when the value of the left-hand side is greater than/equal to/less than the value of the right-hand side, with both sides evaluated at $\widehat{\theta_{E1}}$. Substitution of the value of $\widehat{\theta_{E1}}$ into the equation gives the parameteric condition in the lemma. ■

If $\theta_M \leq \widehat{\theta_M}$, then the entrant enters immediately when θ is in the interval $[\theta_{E1}, \infty)$, where θ_{E1} is determined by equation (26). For $\theta_M < \widehat{\theta_M}$, there are two solutions to the non-linear equation (26). Inspection of the value functions shows that the smaller solution can be ruled out. If $\theta_M > \widehat{\theta_M}$, then the entrant enters immediately. The situation is illustrated in figure A.1. The left-hand side of equation (26) is convex to the origin, with θ plotted on the horizontal axis; the right-hand side is the downward-sloping straight line. The figure shows two possibilities. With the lower convex curve (corresponding to lower values of θ_M), there are two points of intersection; the larger is the relevant solution for θ_{E1}. With the higher convex curve, where $\theta_M = \widehat{\theta_M}$, there is a single intersection—the point of tangency.

The second issue identified above is that the solution to equation (26) may be less than θ_M, contrary to the construction of case 1. Consider the value of θ_M at which $\theta_{E1} = \theta_M$; this value,

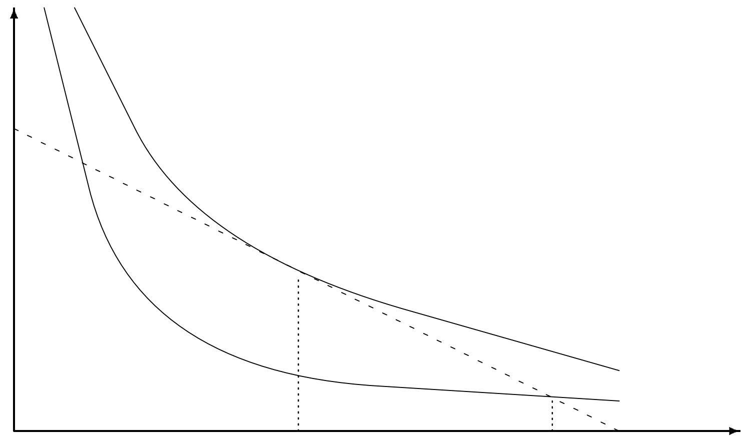

Figure A.1: The solution for θ_{E1}

denoted $\widetilde{\theta_M}$, is the largest solution of the non-linear equation

$$\left(\frac{\alpha+\beta}{\alpha}\right)\left(\frac{\pi(2)}{r-\mu}\right)\theta_X^{\alpha+1}\widetilde{\theta_M}^{-\alpha} + (\alpha+\beta)bS_M(\widetilde{\theta_M}) + (\beta-1)\frac{\widetilde{\theta_M}\pi(2)}{r-\mu} = \beta E. \tag{A9}$$

Necessarily $\widetilde{\theta_M} \leq \widehat{\theta_M}$; and note that $\widetilde{\theta_M} = \theta_X$ is a solution to equation (A9). If $\widetilde{\theta_M} > \theta_X$, then $\theta_{E1} > \theta_M$ for all $\theta_M < \widetilde{\theta_M}$.

These two issues together require that θ_M be sufficiently small. Let

Definition A.1:

$$\overline{\theta_M} \equiv \begin{cases} \widehat{\theta_M} & \text{if} \quad \widehat{\theta_{E1}} \geq \widehat{\theta_M} \\ \widetilde{\theta_M} & \text{if} \quad \widehat{\theta_{E1}} < \widehat{\theta_M}. \end{cases} \tag{A10}$$

With this definition, we can now prove the proposition.

Proof: There are two cases. (i) $\widehat{\theta_{E1}} \geq \widehat{\theta_M}$. In this case, θ_{E1} exists iff $\theta_M \leq \widehat{\theta_M} = \overline{\theta_M}$, from lemma A.1. This in turn is sufficient, given $\widehat{\theta_{E1}} \geq \widehat{\theta_M}$, for $\theta_{E1} \geq \theta_M$. (ii) $\widehat{\theta_{E1}} < \widehat{\theta_M}$. In this case,

$\theta_M \leq \overline{\theta_M} < \widehat{\theta_M}$ is sufficient for θ_{E1} to exist. $\theta_{E1} \geq \theta_M$ iff $\theta_M \leq \widetilde{\theta_M} = \overline{\theta_M}$. ■

Proof of Proposition 4

A_{C1} is continuously differentiable in θ_M; differentiation gives

$$\frac{\partial A_{C1}}{\partial \theta_M} = -\theta_{E1}^{-\beta}\left(\left((\beta-1)-(\alpha+\beta)\left(\frac{\theta_M}{\theta_{E1}}\right)^{\alpha+1}\right)\frac{\partial \theta_{E1}}{\partial \theta_M} + (\alpha+1)\left(\frac{\theta_M}{\theta_{E1}}\right)^{\alpha}\right). \tag{A11}$$

Since $\partial\theta_{E1}/\partial\theta_M < 0$, a necessary condition for $\partial A_{C1}/\partial\theta_M > 0$ at $\theta_M = \theta_X$ is $(\beta-1)/(\alpha+\beta) > (\theta_X/\theta_{E0})^{\alpha+1}$. A necessary and sufficient condition for $\partial A_{C1}/\partial\theta_M > 0$ at $\theta_M = \theta_X$ is

$$\left.\frac{\partial \theta_{E1}}{\partial \theta_M}\right|_{\theta_M=\theta_X} < \frac{-(\alpha+1)\left(\frac{\theta_X}{\theta_{E0}}\right)^{\alpha}}{(\beta-1)-(\alpha+\beta)\left(\frac{\theta_X}{\theta_{E0}}\right)^{\alpha+1}}. \tag{A12}$$

Total differentiation of equation (26) gives

$$\frac{\partial \theta_{E1}}{\partial \theta_M} = \frac{-(\alpha+\beta)\left(\frac{r-\mu}{\pi(2)}\right)\frac{\partial B_{E1}}{\partial \theta_M}\theta_{E1}^{-\alpha}}{(\beta-1)-\alpha(\alpha+\beta)\left(\frac{r-\mu}{\pi(2)}\right)B_{E1}\theta_{E1}^{-(\alpha+1)}}. \tag{A13}$$

At $\theta_M = \theta_X$, the denominator in equation (A13) equals the denominator in equation (A12). Hence a necessary and sufficient condition for the inequality in equation (A12) to be satisfied at $\theta_M = \theta_X$ is

$$(\alpha+\beta)\left(\frac{r-\mu}{\pi(2)}\right)\left.\frac{\partial B_{E1}}{\partial \theta_M}\right|_{\theta_M=\theta_X}\theta_{E0}^{-\alpha} > (\alpha+1)\left(\frac{\theta_X}{\theta_{E0}}\right)^{\alpha}. \tag{A14}$$

From equation (21), this reduces to the inequality in the proposition. With this inequality, $\partial A_{C1}/\partial\theta_M > 0$ at $\theta_M = \theta_X$. By continuity, the result follows.

Proof of Proposition 5

To prove proposition 5, we show for each parameter that the functions on the left-hand side of equations (35) and (36) are monotonic in the parameter.

b: It is clear that equation (35) does not involve b, and that the left-hand side of equation (36) is increasing in b. If $b = 0$, then (clearly) equation (36) cannot be satisfied. If $b = 1$, then, since $(\alpha + \beta)(\Delta\pi/\pi(2) + 1) \geq 1$, equation (36) is satisfied.

$\Delta\pi/\pi(2)$: Manipulation of equation (5) gives the equation

$$\frac{1}{\alpha}\left(\frac{\alpha+\beta}{\beta-1}\right)\left(\frac{\theta_{E0}}{\theta_X}\right)^{-\alpha} + \left(\frac{\theta_{E0}}{\theta_X}\right) = \left(\frac{\alpha+1}{\alpha}\right)\left(\frac{\beta}{\beta-1}\right).$$

Hence the ratio θ_{E0}/θ_X does not depend on $\Delta\pi/\pi(2)$, and so neither does equation (35). The left-hand side of equation (36) is increasing in $\Delta\pi/\pi(2)$. With the restriction that $b > 0$, the result follows.

σ: When $\mu = 0$, $\beta = \alpha + 1$. Manipulation of equation (5) gives

$$(2\alpha+1)\left(\frac{\theta_X}{\theta_{E0}}\right)^{\alpha+1} = (\alpha+1)^2\left(\frac{\theta_X}{\theta_{E0}}\right) - \alpha^2. \tag{A15}$$

The necessary condition in equation (35) requires that the ratio θ_X/θ_{E0} that solves this equation should be such that both sides of the equation are less than α. The right-hand side of equation (A15) equals α when $\theta_X/\theta_{E0} = \alpha/(\alpha+1)$. At this value, the left-hand side equals $(2\alpha+1)(\alpha/(\alpha+1))^{\alpha+1}$; it is straightforward to show that this is less than α for all values of $\alpha \in \mathbb{R}_+$. Hence the solution to equation (A15) is such that equation (35) is satisfied. (This argument is illustrated in figure A.2.) The left-hand side of equation (36) is decreasing in σ (since it is increasing in α and β, and both are decreasing in σ). With the restriction that $b > 0$, the result follows.

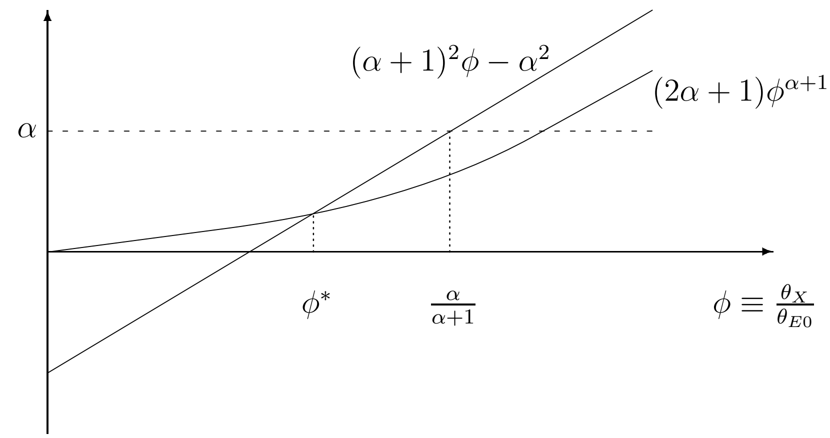

Figure A.2: Satisfying equation (35) when $\mu = 0$

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