BASIC LOGICAL PRINCIPLES FOR ANALYSIS AND SYNTHESIS OF GEOLOGICAL MODELS

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Introduction

Any geological phenomenon results from a set, or an interweaving, of processes, which may sharply differ by their spatial and time scales, nature and origin. Obviously, correct application of modern mathematical and computing methods for modeling such "concoctions" is extremely difficult if ever possible. A large number of models have been developed for a wide range of geological objects and processes but their evaluation and comparative analysis are hindered by the absence of common methodological basis they all could be related to. To formulate it is the motivation for and the general goal of this paper. Formulating it is, in fact, an attempt to strictly locate various mathematical methods (algebraic, geometric, logical) in geology to make the analysis of phenomena, which are difficult to formalize, really formal. The main target of this work is geotectonics.

General Considerations

Modeling can only be based on the knowledge available. Modeled phenomenon is considered a set of processes, being whatever diverse (see above). This automatically rules out the construction of unified model and its study by means of contemporary mathematical physics and existing computer simulation techniques. Hence, our first step should be decomposition of the phenomenon in question into a set of "elementary" (*basic*) processes, which, in our mind, entirely describe the activity of this phenomenon. Their list might be pretty long but necessarily finite, as finite is our reasoning. In addition to finiteness, it should meet three main requirements adopted in classical logic for a system of axioms: *independence, self-consistence and completeness.* The choice of basic processes can indeed be quite subjective. Their list is prone to amendments and changes. But anyway the process of formation of such list (possibly including the reconciliation of standpoints) should precede strict modeling. Once it is formed, let us denote a basic process as B_i, $i = 1...N$, N being the number of basic processes in the list. Each basic process represents a separate mathematical problem. Solution of these problems leads to a set of *basic models*, each corresponding to a basic process, and is the first approximation to the geological phenomenon under study.

From the list of basic models, two different kinds of *working models* (WM) can be developed, the "sum" (1) and the "product" (2) of basic models, correspondingly:

$$WM^s = B_i + B_j + B_k + ... + B_l \quad ; \tag{1}$$

$$WM^p = B_i \otimes B_j \otimes B_k \otimes ... \otimes B_l \quad . \tag{2}$$

Here i, j, k, l are arbitrary integer values from 1 to N. Formula (1) is similar to that suggested by Krumbein and Graybill (1965) for modeling geological systems and processes (models of a coast, of volcanic eruption, etc.) and by Whitten (1964) for geochemical studies of rocks.

To illustrate our general considerations, let us consider the following example.

Example of Decomposition, Addition and Multiplication of Models: Formation of a Sedimentary Basin

The process of formation of a sedimentary basin can be decomposed into the following basic processes:

1. mechanical displacement of substance in the gravity field of the planet,
2. denudation, re-deposition of substance over the surface and sedimentation,
3. migration of light components in the multi-component medium and consolidation of the matter,
4. heat generation and transfer causing changes in the density and rheology of the matter,
5. phase transformations of the matter accompanied by alteration of its mechanical properties;
6. something else can be proposed to (as well as removed from) this list of basic models.

Processes 1-5 show the basis B_i, $i = 1...5$. For each of B_i, the initial conditions and parameters are functions obtainable from solutions of the rest of the basic models. For example, the activity of the surface processes, denudation, re-deposition and sedimentation, depends on topography of the surface, which in turn, is determined by mechanical displacement of the medium. And, vice versa, the rate of subsidence of the basin is dependent on the load made by the mass of the deposited substance. Therefore, solution of an individual basic problem can be used for *"scaling"* of the solution of others. This leads to a "linear model", determined by the "sum" of solutions:

$$WM^s = B_1 + B_2 \,.$$

However, if we intend to consider the deformation processes and those of denudation and sedimentation in their interaction, it is necessary to develop a mathematical model, represented by a system of interrelated equations. Such "nonlinear" model would look like

$$WM^p = B_1 \otimes B_2 \,.$$

This illustrates the sense of operations of "addition" and "multiplication" of models. The same applies to any subset of basic models from the considered set (list). The quantitative estimation of each of the models is the quantitative estimation of the solution of the corresponding mathematical problem in a normalized space of functions.

Vertical displacement $w(x, y)$ of the surface (model B_1) can be described by the equation pertaining to thin flexible film on a nonviscous substrate:

$$D\nabla^4 w(x, y) = -\rho_t g_0 H(x, y) - \rho_m g_0 w(x, y)\,, \qquad (3)$$

H is the topography of the surface, ρ_t, ρ_m are densities of topography and mantle matter, correspondingly, g_0 is the acceleration due to gravity.

Evolution of the surface due to denudation and sedimentation processes (model B_2) can be described by the equation, which is similar to the equation of diffusion:

$$\frac{\partial h}{\partial t} = div(k(x, y) \cdot grad\,(h(x, y))) \qquad (4)$$

The observed topography is described as follows:

$$h = H + w, \qquad (5)$$

$$\frac{\partial (H + w)}{\partial t} = div\,(k(x, y) \cdot grad\,(H + w))\,. \qquad (6)$$

The "product" of two models B_1 and B_2 is the solution of the *system* of two equations: (3) and (6).
The "sum" of these two models is the solution of equation (3) for given H and solution of equation (6) for given w; w can be taken as the solution of equation (3).
The representations (3) and (6) of the corresponding basic models are not unique. There are a number of different mathematical models that can be constructed for simulation of the process under study. The choice of a proper one is another challenge for mathematical modeling which resides in more specific fields of knowledge, like mechanics, chemistry, etc.

Nonlinearity

The assumption that each of the basic processes is acting *independently* from others means that we have N independent basic models and N solutions of the corresponding mathematical problems. The next step to modeling a geological phenomenon is accounting for interaction of the processes involved. It is worth mentioning here that interaction of *all* basic processes is optional. Let us assume that they interact only in pairs. Then a new composite model can be written as a "multiplication" of two basic models:

$$B_{ij} = B_i \otimes B_j \,.$$

Mathematically, the model B_{ij} should be represented as a system of equations of the mathematical physics, whose solution will allow us to obtain the recurrent approach to the modeled process. One can develop different working models WM^p by the formula (2) from the elements of the basis B_i. We can act in this way a number of times until we "reach" the initial phenomenon, which is represented by the product:

$$WM^p = B_1 \otimes B_2 \otimes B_3 \otimes \ldots \otimes B_N \,.$$

Formula (1) can be considered as a "linear" approach to modeling of the geological processes. Within the framework of the formalism developed here the "quadratic" forms (models) $B_{ij} = B_i \otimes B_j$ can be considered as models of "nonlinear" geodynamics. The "nonlinearity" is interpreted as the "multiplication" of models $B_{ij} = B_i \otimes B_j$ though one should keep in mind that mathematical models of some basic processes B_i can be initially nonlinear.

Catastrophes

Estimation of the solution on a normalized space of functions can be considered a quantitative characteristic β_i of a model. Thus, each working model can be represented as a point ($\beta_1, \beta_2, \ldots, \beta_N$) in N-dimensional Euclidean space and thus the distance between these can be determined. If it is stipulated that coefficients β_i should satisfy a restriction, e.g., their sum is equal to the constant E, then each working model can be considered as a point on the hyperplane:

$$\sum_i \beta_i = E \tag{7}$$

E can be, for instance, the total energy of the geological process under consideration. The basis B_i is considered as a set of the *evolutionary* processes. Therefore, the solution of the corresponding mathematical problems and "coordinates" $\{\beta_1, \beta_2, \ldots, \beta_N\}$ of the working model can be considered as the continuous function of time, and the evolution of WM can be shown graphically as a continuous curve $\beta(t) = \{\beta_1, \beta_2, \ldots, \beta_N\}$ in the space, or on the hyperplane (7); t is a geological time.

A catastrophic event can change the model under study considerably – to bring to a stop some processes and introduce other ones. Probably, in each case, setting of the new problem of modeling with a choice of the new basic functions and new initial conditions will be most expedient. Geometrically, the moment of a catastrophe is marked by break in continuity of the evolutionary curve $\beta(t)$.

How Many Working Models Can Ever Be Developed?

As far as each WM^p is a non-arranged set of *n* models (*n=1,2,…,* N) from N possible variants, their total number is

$$C_N^1 + C_N^2 + ... + C_N^9 + C_N^N - 1 = 2^N - 1$$

The same number of working models can be constructed for the "sum" WM^s of basic models B_i .

Thus, we have a broad range of the models for the geological phenomenon in question and are able to enumerate and consider each of them.
For instance, if a phenomenon is decomposed into four basic models, the total number of "products", excluding basic models B_i , is 11.

Discussion and Conclusions

The suggested approach is based on decomposition of the modeled geological phenomenon in a set of basic (elementary) processes, followed by synthesis of models starting from the most simple up to the most complicated ones. The latter gives a representation of entire phenomenon in terms of accepted basic processes. Comparison of the models and estimation of the distances between them result from the comparison of the quantitative estimations of the corresponding mathematical solutions in a normalized space of functions.
This methodology allows us to formulate a number of recommendations, which are desirable to be used for mathematical modeling of the geological processes.

1. It is necessary to make the list of basic processes B_i , which are constituents of the initial phenomenon.
2. Mathematical models should be formulated for each basic process in such a way that the functions of the rest of the problems searched for should be included in these models as the parameters given.
3. One can construct mathematical models taking into account the interaction of two, three and more of basic processes, that is, find products of the models $WM^p{}_{ij...k} = B_i \otimes B_j \otimes ... \otimes B_k$ and their solutions on condition the complexity of the problems allows us to do this.
4. In the case further complication and solution of WM^p models is not possible, one can construct and to analyze the working model, which is the "sum" of the "product" models: $WM^s = \sum WM^p{}_{ij...k}$
5. The graph should be compiled reflecting all the steps of construction and analysis of models**.** It can be the starting point for further modeling.

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References

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