Fitting covariate models to extreme value data: an example from the formal investigation into the loss of the Derbyshire

RSS / ESSG short course

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1 The bulk carrier MV Derbyshire

- caught in Typhoon Orchid on the 9th September 1980
- sank 350 miles south east of Japan
- all 44 people on board died
- no mayday was received
- largest UK ship ever lost at sea

2 The investigation

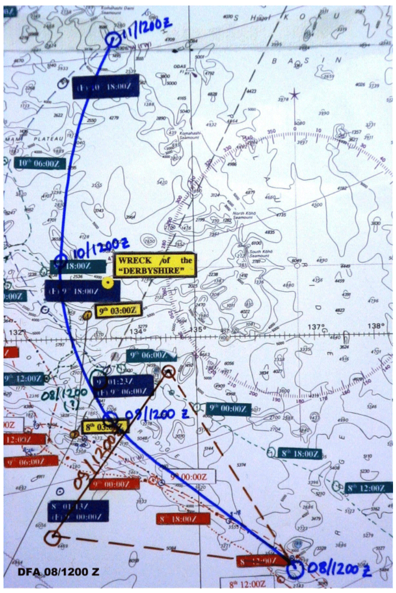
- the wreckage was found in 1994
- 2.5 miles under water
- 200 hours of video and 135,000 photos of the wreck
- enquiry was heard by Mr. Justice Colman during April – July 2000

Our role in the investigation:

- focus on the risk of waves on hold 1 (at the front of the vessel) exceeding the collapse pressure of 42 kPa

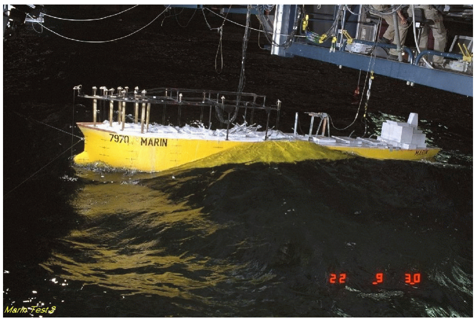
3 The data

- know where ship sank
- satellite data gives Typhoon weather information
- wave experts *hindcast* wave conditions



4 The data

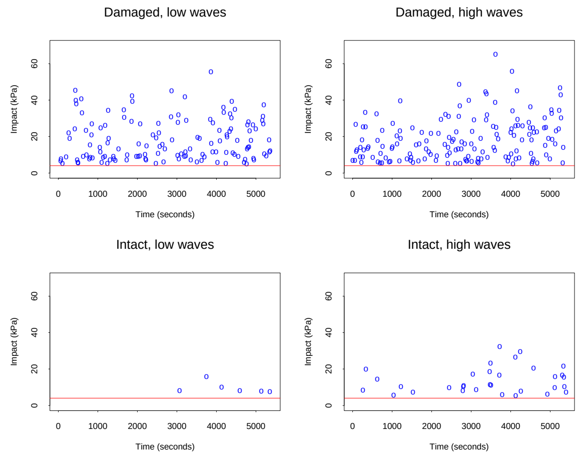
- Marine Research Institute, Netherlands (MARIN)
- replica of the Derbyshire
- range of ship and wave conditions in a test tank
- sensors recorded wave impacts on Hold 1



5 The data

peaks above 5 kPa separated by at least 8 seconds

- range of wave and ship conditions
- influences number and size of impacts



24 tests total to cover range of ship and wave conditions

6 Model for impact distribution

Data are independent threshold exceedances.

Try Generalised Pareto family – GPD(σ, ξ):

$$F_u(x) = 1 - \left\{1 + \xi\left(\frac{x-u}{\sigma}\right)\right\}^{-1/\xi}$$

for $x > u$ and $1 + \xi(x-u)/\sigma > 0$.

First step – fit GPD separately to each test:

1. determine suitable threshold
2. maximum likelihood
3. validate model fit

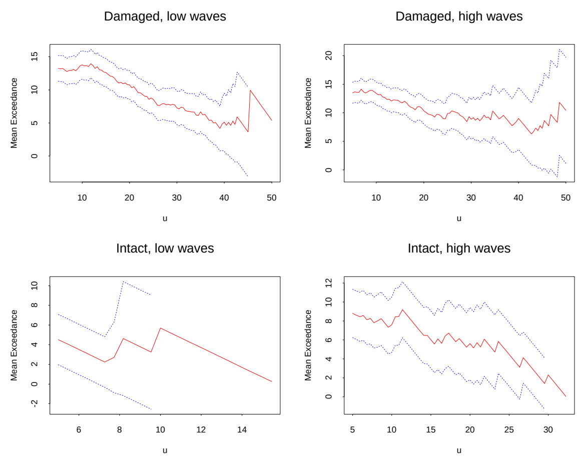
7 Threshold choice – Mean residual life plots

If $X - u \mid X > u$ follows a GPD(σ, ξ)

then (if $\xi < 1$) for all $u^* > u$,

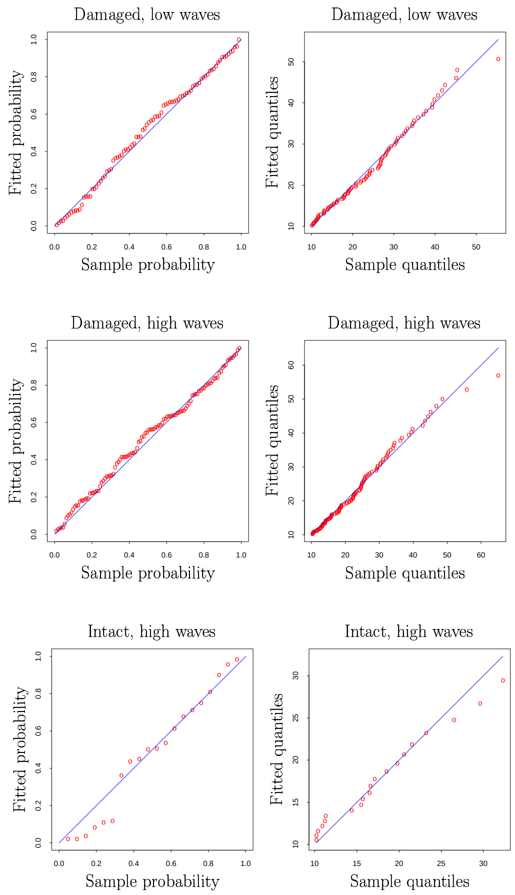
$$E(X - u^* \mid X > u^*) = \{\sigma + \xi(u^* - u)\}/(1 - \xi)$$

This is linear in u^* with gradient $\xi/(1-\xi)$.



Choose threshold $u = 10$kPa.

8 Maximum likelihood – validate model fit



9 Maximum likelihood

Fit GPD to threshold exceedances.

Data	n	σ	(s.e.)	ξ	(s.e.)
Damaged, low	88	18.8	(2.3)	-0.38	(0.07)
Damaged, high	110	16.4	(1.9)	-0.23	(0.07)
Intact, low	1	–	–	–	–
Intact, high	20	10.65	(3.6)	-0.37	(0.27)

- different numbers of points
- different scale parameters
- evidence of same shape parameter?

Remember:

- shape parameter most difficult to estimate
- beneficial to share information across data sets

...must check whether data support this.

10 Fitting common shape parameter

Exceedances $X - u$ from ith test follow GPD(σ_i, ξ)

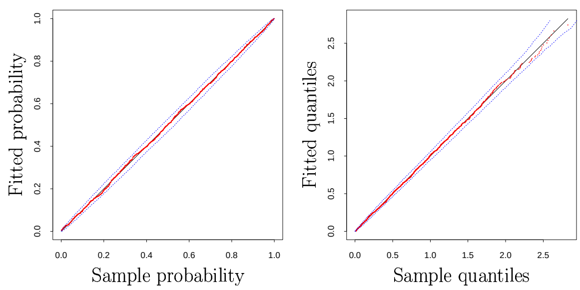
Combine data from ALL tests to give 1180 points

Standardised exceedances:

$$\frac{X - u}{\sigma_i}$$

will then follow GPD$(1, \xi)$.

PP and QQ plots on common scale:



Common shape parameter estimate $\xi = -0.33$

Different scale parameters σ_i depend on conditions of ith test.

11 Think about original problem again...

AIM:

- model distribution of impacts on Hold 1
- range of weather conditions experienced during Typhoon Orchid
- range of boat conditions, as flooding state of boat deteriorated

PROGRESS SO FAR:

- have GPD model which fits test data well
- one shape parameter
- separate scale parameter for each test (= 24 parameters)

CAN WE ANWSER THE QUESTION YET?

- test weather conditions don't cover all weather conditions in Typhoon
- test boat conditions don't cover all boat conditions we want to investigate

12 Using covariates...

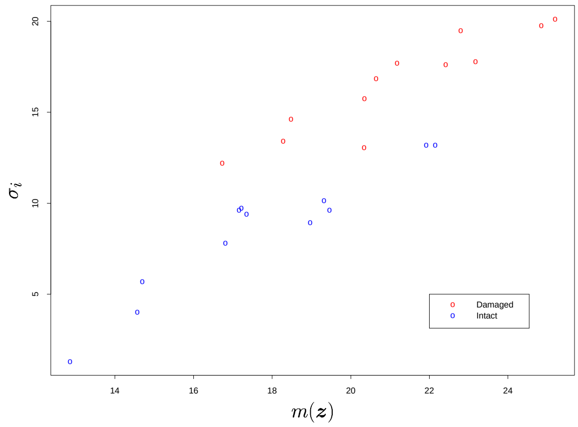
Differences between impact distributions due to:

- weather conditions
- boat conditions

Let $\boldsymbol{z}$ represent all this information.

Wave theory then gives us the summary variable:

$$m(\boldsymbol{z}) = \text{ expected value of impacts above 10 kPa}$$



13 Using covariates...

So σ depends on

- $m(\boldsymbol{z})$
- damaged / intact

In fact – the real variable we should look at is not just a damage *indicator* but

$$\mathrm{F}(\boldsymbol{z}) = \text{freeboard}$$

distance from fore deck to still water level.

Based on picture, postulate model:

$$\sigma_i = \sigma(\boldsymbol{z}_i) = a_0 + a_1 F(\boldsymbol{z}_i) + a_2\{m(\boldsymbol{z}_i) - u\}.$$

where $\boldsymbol{z}_i$ are boat and storm conditions in ith test.

(try range of models for best fit and parsimony)

14 Fitting the covariate model

Likelihood with different scale parameters
(no covariates yet!):

$$L(\boldsymbol{\sigma}, \xi) = \prod_{i=1}^{24} \prod_{j=1}^{n_i} \frac{1}{\sigma_i} \left\{ 1 + \xi \frac{(x_{i,j} - u)}{\sigma_i} \right\}^{-(1+1/\xi)}$$

- 1 shape parameter
- 24 scale parameters
- maximised numerically

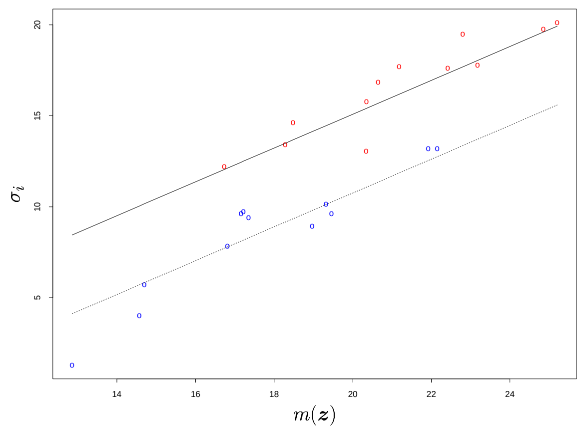
To fit the covariate model, replace each σ_i with

$$\sigma_i = \sigma(\boldsymbol{z}_i) = a_0 + a_1 F(\boldsymbol{z}_i) + a_2\{m(\boldsymbol{z}_i) - u\}.$$

- 1 shape parameter
- three other parameters
- scale parameters are functions of covariates
- maximised numerically

15 Using covariates...

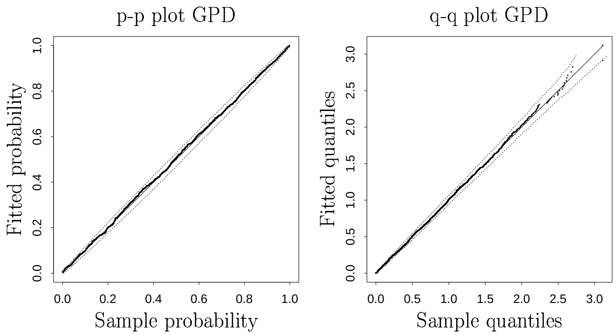
$$\sigma(\boldsymbol{z}) = a_0 + a_1 F(\boldsymbol{z}) + a_2\{m(\boldsymbol{z}) - u\}.$$



16 Validate model fit

Has reduction in number of parameters worsened fit of model?

Use formal tests (likelihood ratio etc...)



Fit still excellent,

Shape parameter $\hat{\xi} = -0.30$

- light tail
- impact distribution has finite upper end point
- value of upper end point depends on covariates through scale parameter $\sigma(\boldsymbol{z})$

17 Benefits of covariate model

No longer have one σ_i for each test, instead

- three parameters a_0, a_1 and a_2,
- models scale of impact distribution as function of covariates

Model has physical interpretation:

- sea conditions *worsen* – σ increases
- freeboard *decreases* – σ increases

Can also now use model to predict impact distribution for all Typhoon conditions:

- calculate $m(\boldsymbol{z})$ for all Typhoon conditions
- calculate $F(\boldsymbol{z})$ for deteriorating state of boat

18 Maximum impact in any hour

Number of impacts is also random.

Wave theory gives us further summary of sea and ship conditions:

$$\lambda(\boldsymbol{z})$$

expected number of impacts > 10 kPa on Hold 1 in hour with conditions $\boldsymbol{z}$.

We assume number of such impacts is

- Poisson with mean $\lambda(\boldsymbol{z})$

The distribution of the maximum impact C_j in hour j, given covariates $\boldsymbol{z}_j$ is then:

$$\boxed{\Pr\{C_j \leq x \mid \boldsymbol{z}_j\} = \exp[-\lambda(\boldsymbol{z}_j)\{1 - F_u(x;\boldsymbol{z}_j)\}]}$$

for $x > u$.

Here $F_u(x;\boldsymbol{z}_j)$ is distribution of impacts > u kPa.

We have modelled $F_u(x;\boldsymbol{z}_j)$ as GPD with parameters $\sigma(\boldsymbol{z}_j)$ and ξ.

19 Maximum impact in storm

The maximum impact over d consecutive hours has distribution function

$$\prod_{i=1}^{d} \Pr(C_j \leq x \mid \boldsymbol{z}_j).$$

This assumes:

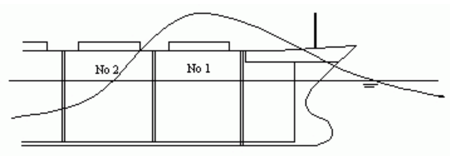
- wave and ship conditions (represented by $\boldsymbol{z}_j$) are stationary over any hour j
- the C_j are conditionally independent given the $\boldsymbol{z}_j$

Since $m(\boldsymbol{z})$, $\lambda(\boldsymbol{z})$ and $F(\boldsymbol{z})$ were available for the evolving typhoon conditions we can estimate this distribution for each hour of the storm.

20 Risk estimates

The estimated risk of hold 1 receiving an impact above 42 kPa during the typhoon

- is negligible if no initial flooding has occurred
- varies between zero and one over the range of flooding scenarios

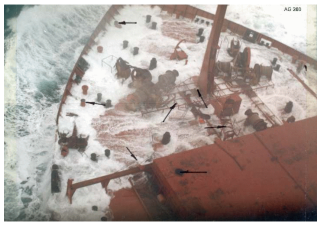


Initial flooding	Waves	Pr(impact>42 kPa) (95% conf int)
None	Hindcast	0.00 (0.00, 0.00)
	10% higher	0.01 (0.00, 0.29)
Stores	Hindcast	0.00 (0.00, 0.05)
Deep tank		0.00 (0.00, 0.03)
Ballast tank		0.71 (0.38, 0.96)

Likelihood based confidence intervals let us tell whether the risk estimates are truly different from each other.

21 Summary / discussion

- GPD gives excellent fit to data recording excesses over thresholds
- more sophisticated modelling is needed as prediction is required for scenarios not represented by the test data
- covariates provide explanation of differences between tests
- likelihood framework natural for fitting complex models of this type
- confidence intervals vital to show uncertainty in risk estimates



22 Investigation outcome

The Judge's report attributes the loss to

- initial damage to ventilation and air pipes caused by sustained wave loading
- flooded various of the vessel's cavities and reduced the freeboard
- increasing impacts to hold 1 and finally causing the hatch cover to fail
- hold 1 would then have flooded rapidly
- damage imperceptible from bridge, at stern
- flooding of holds 2 and 3 would follow
- ship would then inevitably be lost

More information at `www.mv-derbyshire.org.uk`