Collapse of the thermohaline circulation
Emulators
Some Experiments
Towards Trajectories

How we might put this into practice: the probability that the Atlantic circulation collapses

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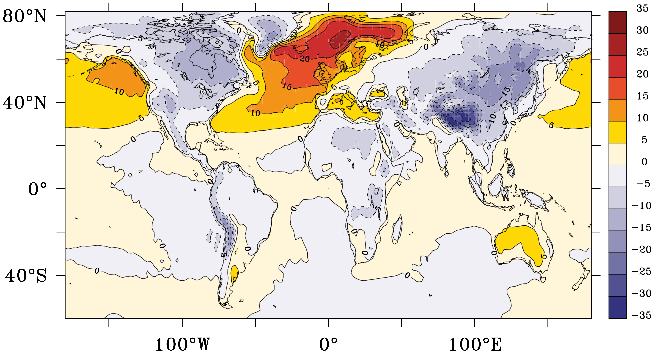
The thermohaline circulation

North West Europe is warm compared to similar latitudes



The thermohaline circulation

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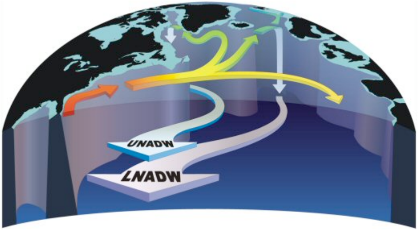
- This is because heat is transported N in the Atlantic
- This heat comes, not from the Gulf Stream but from the Thermohaline Circulation



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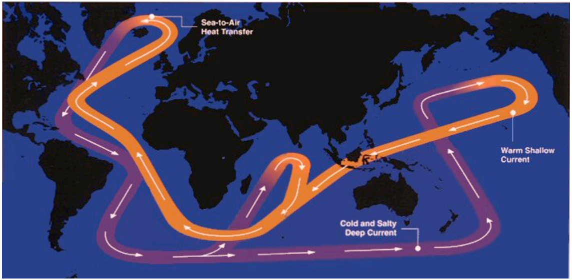
- Cold salty water sinks in the North and flows south at depth
- Warm, fresh water is brought north



The Big Question

- What is the probability that the Meridional Overturning Circulation in the North Atlantic (MOC) will collapse by 2100?
- What is the probability that the Meridional Overturning Circulation in the North Atlantic (MOC) in a ensemble of different models will collapse by 2100?
- What is the probability that the Meridional Overturning Circulation in the North Atlantic (MOC) in a particular model will collapse by 2100?

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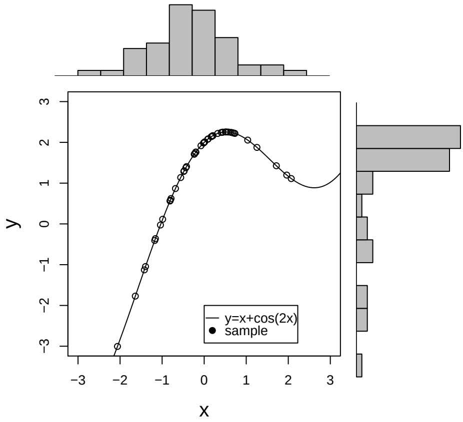
Monte Carlo Estimate



MC error is proportional to $\frac{1}{\sqrt{n}}$

The Emulator

- An emulator is statistical approximation to the computer model
- Or an encapsulation of our beliefs about the model
- A Gaussian Process that models a non-linear function as a mean function plus a stochastic process
- All marginal and conditional distributions are Gaussian; with mean function $\mu(x)$ and a covariance function $\rho(x_1, x_2)$
- We not only get a estimate of the climate model output at our input, x, but also a measure of its uncertainty

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Some Maths

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The Prior

$$\mu(x) = h(x)^T \beta$$

h(.) is a known vector of regressor (or basis) functions
e.g. $h(x)^T = (1, x, x^2)$
β is a vector of unknown parameters

$$\rho(x_1, x_2) = \sigma^2 c(||x_1, x_2||)$$

$$c(x_1, x_2) = exp\left(-\frac{||x_1, x_2||^2}{\theta}\right)$$

$$p(\beta, \sigma^2) \propto \sigma^{-2}$$

The Posterior

$\eta(x) \sim t_{n-q}$

$$E(\eta(x)) = h(x)^T\beta' + t(x)^T A^{-1}(y - H\beta')$$
$$\beta' = \left(H^T A^{-1} H\right)^{-1} H^T A^{-1} y$$

H is the matrix $\{h(x_1), \cdots, h(x_n)\}^T$

$$t(x) = \{c(x, x_1), \cdots, c(x, x_n)\}$$

A is the matrix $\{c(x_i, x_j)\}$
And there are similar, but more complex, expressions for the variance

Smoothness

- θ is the smoothness or scale of the GP
- θ is not included in our posterior because it isn't included in the Bayesian solution
- Use maximum posterior (likelihood) or cross validation to estimate θ
- This is non-trivial
- Does it matter?

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Example

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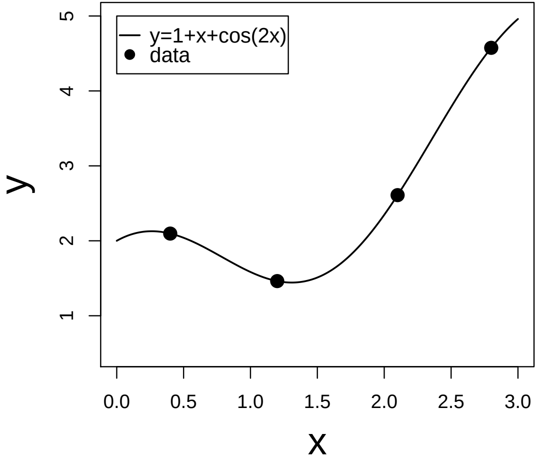

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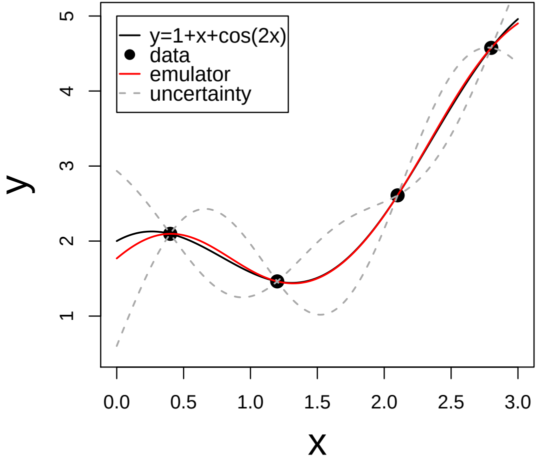

Nuggets

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- We have fitted the model $\eta(x) = \mu(x) + \phi(x)$
- This goes exactly through each data point
- We could fit $\eta(x) = \mu(x) + \phi(x) + \epsilon$
- where ϵ is a 'nugget' term $\sim N(0, \sigma_\epsilon^2)$

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Why include a nugget

- We have a *strong* prior that the emulator should be equal to the climate model at these points
- BUT ...
- With a small nugget estimate of the scale parameter is much more stable
- Although we no longer reproduce our deterministic inputs in general we get a smaller prediction error for points not in the calculation
- It is a way of dealing with non-active variables

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Active and non-active variables

- If we have a large number of input variables we can divide them into
- Active variables - which matter
- Non-active variables - which don't
- e.g. In a problem with 20 input variables we may find that 5 of the variables in the μ term explain 90% of the variation. We can now reduce our problem to 5 active variables + 15 non-active variables
- Model the effect of the non-active variables as a nugget

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Estimating the probability

- Specify an uncertainty distribution for each model parameter
- Run a designed set of model runs to span this parameter space
- Estimate the parameters of the emulator
- Sample a large number (thousands) of points from the uncertainty distributions
- Evaluate the emulator at each of these points.
- Estimate the pdf and calculate the probability of being less than a specified value

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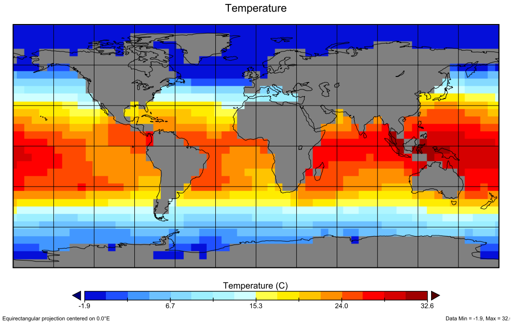
GENIE aka C-GOLDSTEIN

- GENIE is a framework of intermediate complexity climate models
- GENIE-1 has a thermocline ocean model component and an energy balance atmosphere
- We are not going to consider versions with dynamic atmospheres or explicit bio-geochemistry

GENIE-1 grid

- Atmosphere - ocean - sea ice
- Intermediate complexity - 64 x 32 x 8 (36 x 36 x 8)
- (pretty) fast - 100 years in a few hours



GENIE-1

- GENIE has *about* 15 (+2) unknown input parameters

Inputs

Parameters

- Ocean viscosity
- Moisture transport
- Climate Sensitivity ...

Forcings

- Carbon dioxide
- Greenland Melting ...

Outputs

- Ocean/air temperature
- Rainfall
- Salinity
- Heat/moisture fluxes
- Ocean currents
- Max. Atlantic overturning circulation

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The Training Experiment

- To generate the training set we run the model in a designed experiment
- This ensemble is not designed to give a realistic climate but to span parameter space
- The most common design is the Latin Hypercube
- In our experiments we have one hundred member ensembles for training

The Latin hypercube

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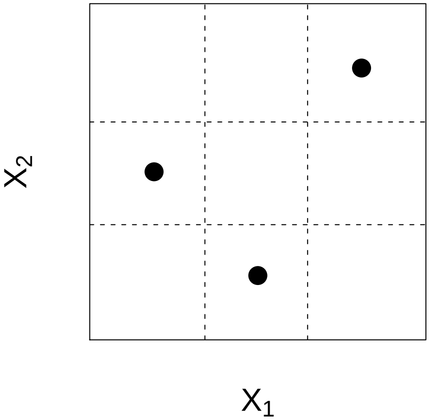

Not all Latin hypercubes are equal

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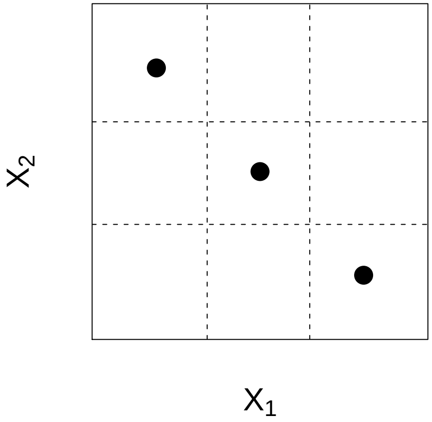
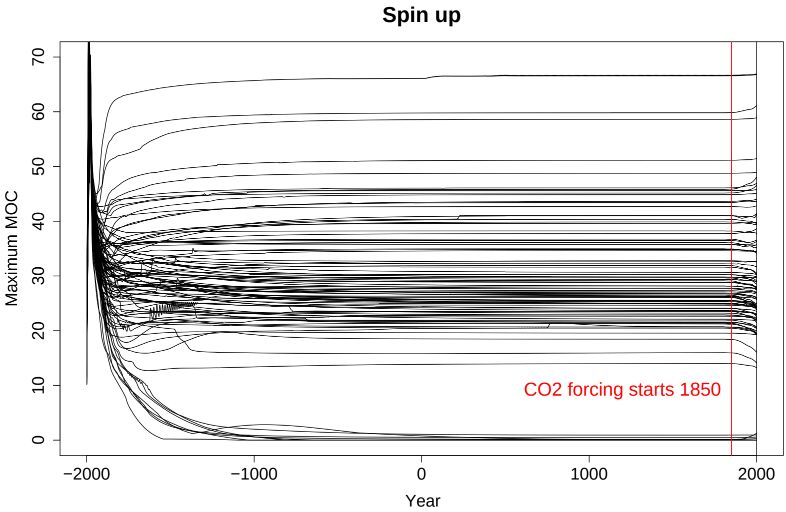


Spin-up of GENIE-1



GENIE projects the Future

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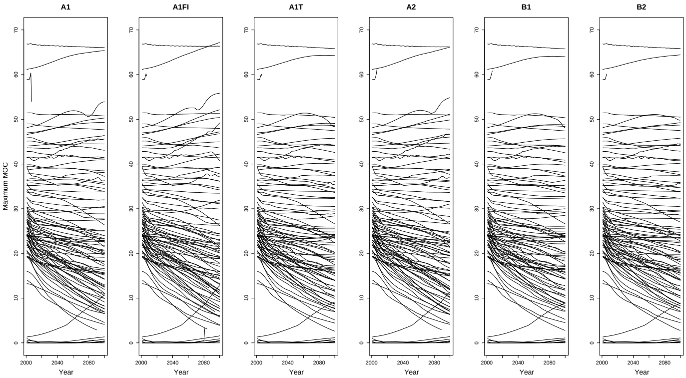

The Avoiding Dangerous Climate Change book experiment

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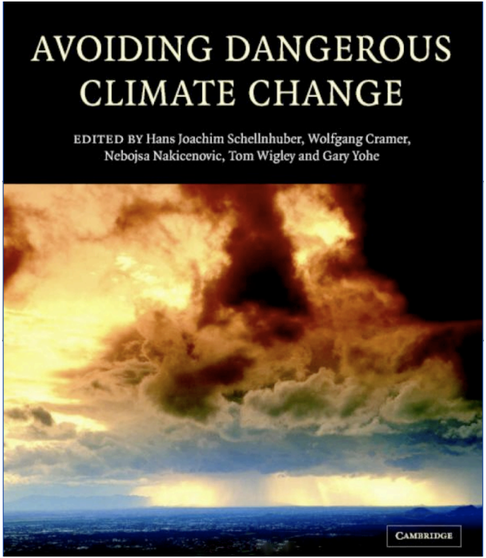

The Avoiding Dangerous Climate Change book experiment

A previous experiment:

- defined collapse as < 5 Sv at 2100
- Found high probability of collapse around 30% - 40% depending on the scenario
- Used GENIE 36 x 36 x 8

Table 2. Probability of Atlantic overturning falling below 5 Sv by 2100

Uncertainty Case	SRES scenario A1	A2	B1	B2	A1FI	A1T
default uncertainty						
Case 1a	0.37	0.38	0.31	0.32	0.43	0.32
Case 1b	0.38	0.40	0.30	0.31	0.46	0.31
doubled uncertainty in Climate sensitivity						
Case 2a	0.37	0.38	0.33	0.33	0.43	0.33
Case 2b	0.39	0.40	0.31	0.32	0.46	0.32
doubled uncertainty in Atlantic-Pacific moisture flux						
Case 3a	0.37	0.38	0.32	0.33	0.43	0.33
Case 3b	0.40	0.40	0.30	0.30	0.46	0.32
doubled uncertainty in CO_2 uptake						
Case 4a	0.38	0.38	0.31	0.32	0.44	0.33
Case 4b	0.38	0.39	0.31	0.31	0.44	0.32
doubled uncertainty in Greenland melt rate						
Case 5a	0.37	0.38	0.31	0.32	0.43	0.32
Case 5b	0.38	0.39	0.30	0.32	0.45	0.32

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Don’t panic

- This version of GENIE (36 x 36 x8) tends to collapse
- Higher resolution versions (64 x 32 x 8) don’t
- Wider Atlantic?

Multivariate outputs

We would like to:

- We want to predict high dimensional output from GENIE.
- We only emulate a single output variable (at present)
- We can perform an PCA analysis of model output across the ensemble.
- We can predict a few principal components, each with a separate emulator.

Maximum MOC ensemble

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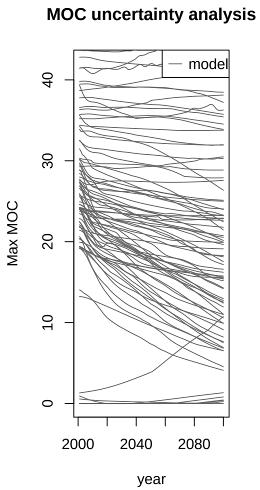

Maximum MOC uncertainty analysis

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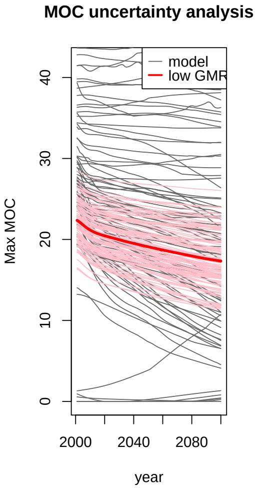

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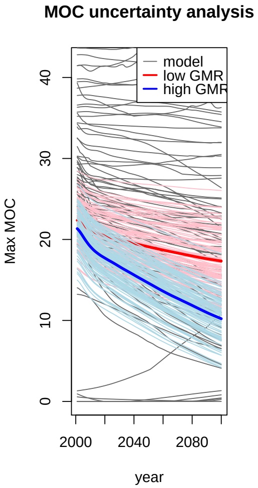

Calibrating the model

- So far we have not included any data in our analysis
- Only 5 estimates of the strength of the MOC available (1957, 1981, 1989, 1998, 2004)
- 'Through the window'

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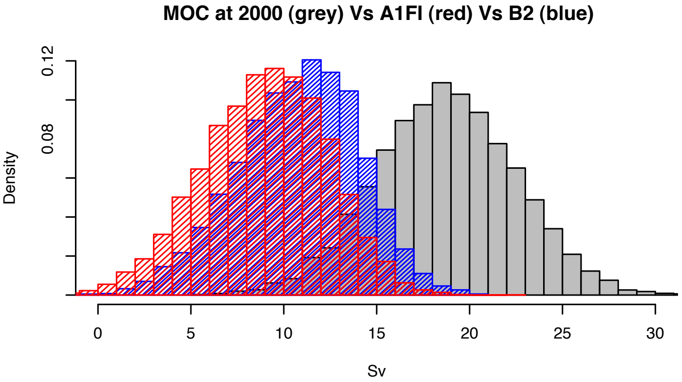
MOC at 2000 (grey) Vs A1FI (red) Vs B2 (blue)
Density
0.12
0.08
Sv
0
5
10
15
20
25
30

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$$d = \eta + \delta + \epsilon$$

$$\begin{aligned} d &= \text{data} \\ \eta &= \text{climate model} \\ \delta &= \text{model discrepancy} \\ \epsilon &= \text{error} \end{aligned}$$

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Conclusions

- We have developed a basic methodology to estimate probabilities of low probability/ high impact events such as the collapse of the THC.
- We are extending these methods e.g. climate trajectories, calibration.
- So far these are partial solutions as we are only working with single models
- We now need to apply these to a variety of models; especially to GCM's
- Need to relate our models to reality - we are interested in the *real* world not the *model* world

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- We are extending these methods e.g. climate trajectories, calibration.
- So far these are partial solutions as we are only working with single models
- We now need to apply these to a variety of models; especially to GCM's
- Need to relate our models to reality - we are interested in the *real* world not the *model* world