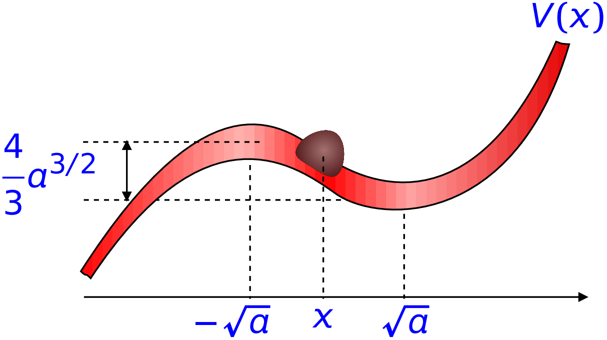
Outline — 2nd part

- Noise induced escape near tipping
- Estimates of normal form parameters from time series

Noise induced escape

overdamped particle in a well

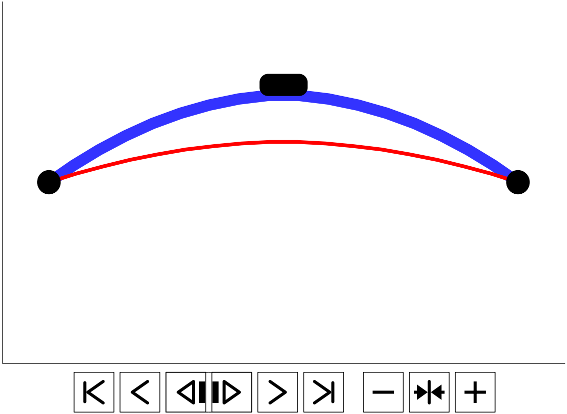


$$\frac{\mathrm{d}}{\mathrm{d}t}x = -V'(x) = a - x^2 \,\textbf{+ noise}$$

Mechanical caricature

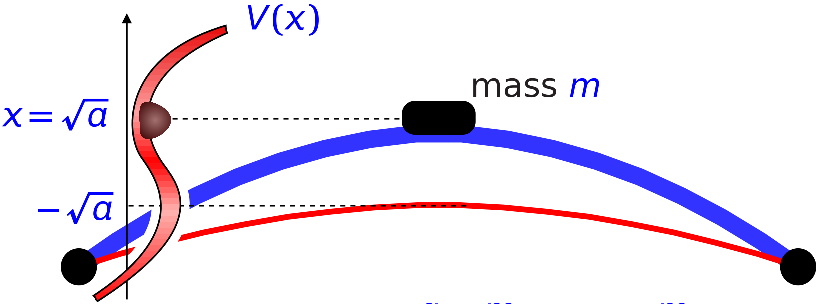
of positive feedback

- squishy beam, clamped and loaded with gradually increasing mass m



Mechanical caricature

of positive feedback



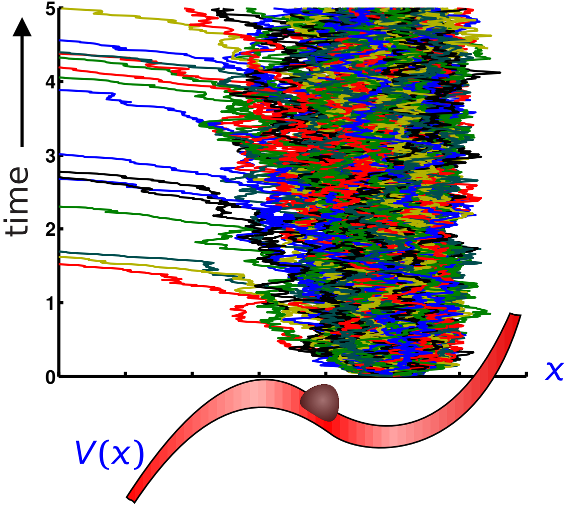
$$a \sim m_{\mathrm{critical}} - m$$

$$\frac{\mathrm{d}}{\mathrm{d}t} x = -V'(x) = a - x^2 \textbf{ + noise}$$

University of Portsmouth

Noise induced escape

Fixed well depth, Noise amplitude $\sigma > 0$



University of Portsmouth

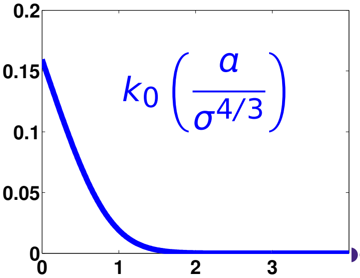
Noise induced escape

Fixed well depth, Noise amplitude $\sigma > 0$

- $a \gg \sigma$ (barrier $\gg$ noise amplitude):
  Kramers' escape rate

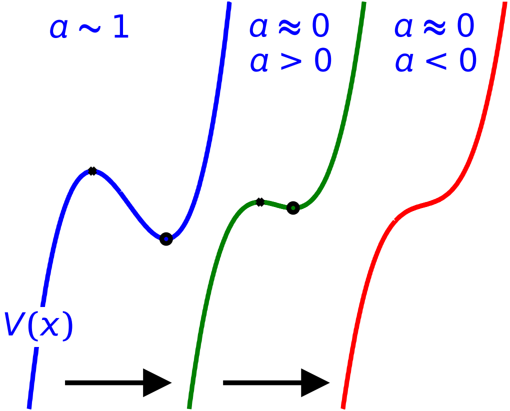
$$k_0\left(\frac{a}{\sigma^{4/3}}\right) \sim \frac{2}{\pi}\sqrt{\frac{a}{\sigma^{4/3}}}\exp\left(-2\sqrt{\frac{a}{\sigma^{4/3}}}\right)$$

- $a \sim \sigma, \quad a \leq \sigma$
  linear boundary value problem for probability density $p(x)$
  Fokker-Planck



Noise induced escape

Shrinking well: $\dfrac{\mathrm{d}}{\mathrm{d}t}a = -\varepsilon$**, Noise amplitude** $\sigma > 0$

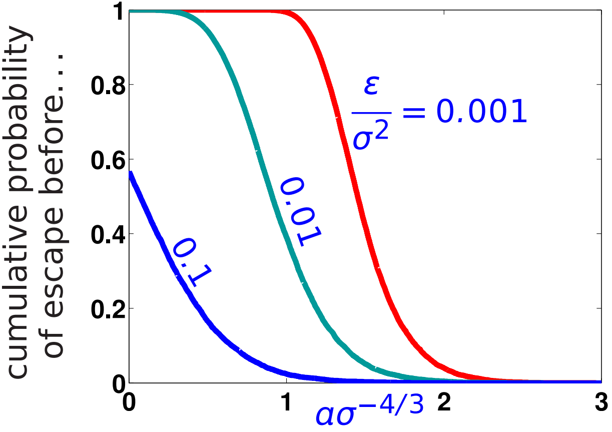


University of Portsmouth

Noise induced escape

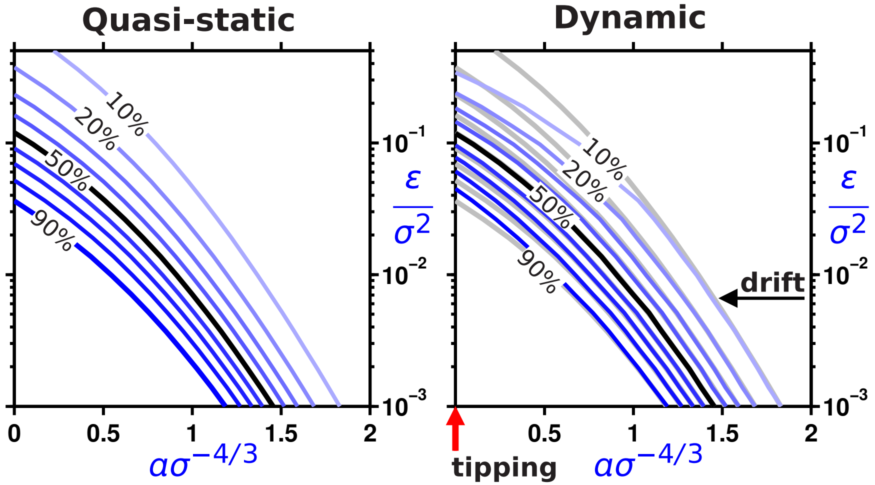
Shrinking well: $\frac{\mathrm{d}}{\mathrm{d}t}a = -\varepsilon$, **Noise amplitude** $\sigma > 0$

start from $a_0 \gg \sigma^{4/3}$



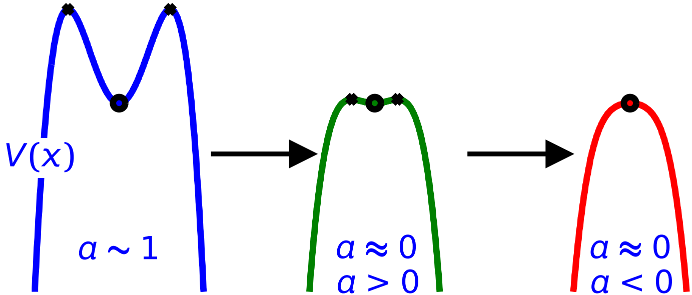
University of Portsmouth

Cumulative escape probabilities



University of Portsmouth

Related work & References

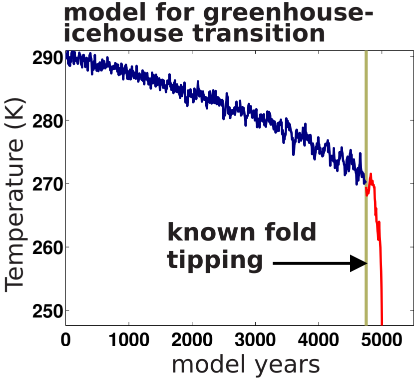


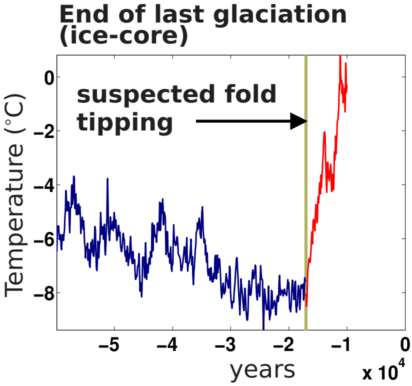
Early or delayed escape, under-damped

Miller/Shaw	$\Longleftarrow$	Engineering
Berglund/Gentz	$\Longleftarrow$	Mathematical theory
Kuske	$\Longleftarrow$	Fokker-Plank equations

Estimate from time series

2 Examples (Dakos *et al*)



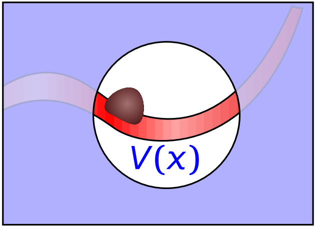


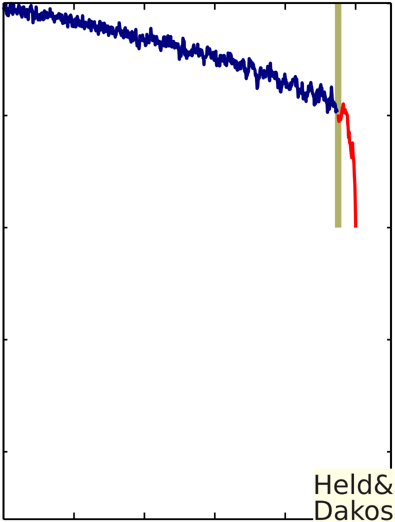
Estimation procedures:

- Autocorrelation coefficients (Held/Kleinen)
- Detrended fluctuation analysis (Livina/Lenton)

University of Portsmouth

Estimate from time series





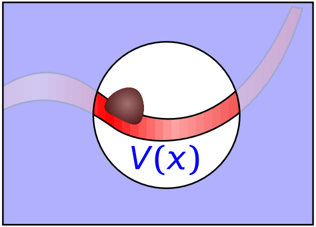
measurements noise

$$x_{n+1} = \mathbf{c}\, x_n + \sigma\, \eta_n$$

ARC(1)

Held&Kleinen
Dakos *et al*

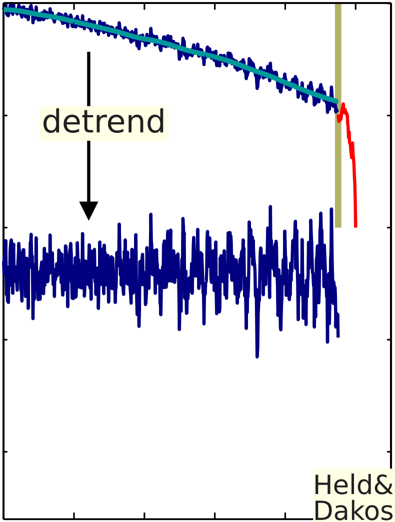
Estimate from time series



measurements noise

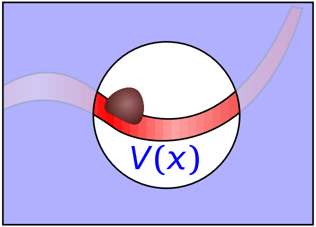
$$x_{n+1} = \mathbf{c}\, x_n + \sigma\, \eta_n$$

ARC(1)



Held&Kleinen
Dakos *et al*

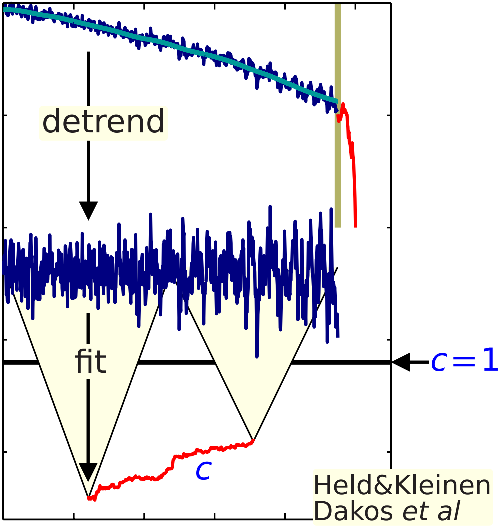
Estimate from time series



measurements noise

$$x_{n+1} = \mathbf{c}\, x_n + \sigma\, \eta_n$$

ARC(1)



Estimate from time series

Relation of estimates from AR(1) model to normal form quantities

$$x_{n+1} = \mathbf{c} x_n + \sigma \eta_n$$

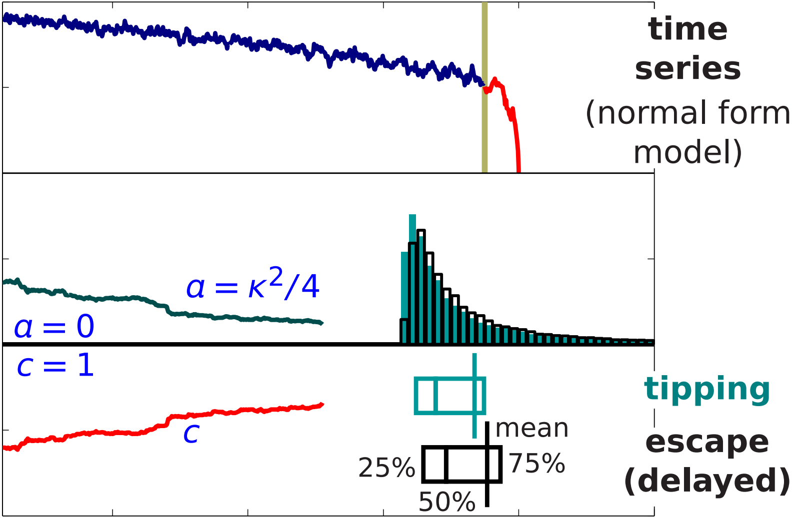
time step of linearized stochastic process

$$x_{n+1} = (\mathbf{1} - \kappa \mathbf{\Delta t}) x_n + \sigma \sqrt{\mathbf{\Delta t}}\, \eta_n$$

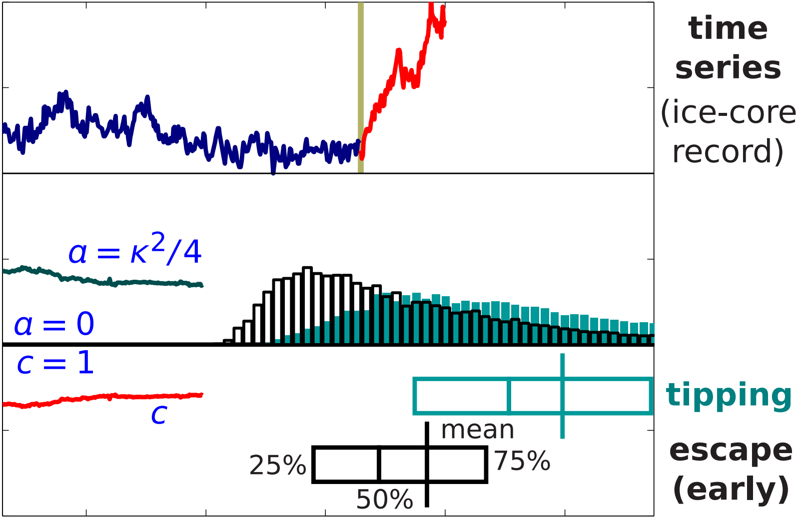
- κ linear decay rate per Δt
- normal form parameter $\alpha = \frac{\kappa^2}{4}$
- scaling $\frac{\text{avg slope of } x_{\text{trend}}}{\text{avg slope of } \kappa}$

University of Portsmouth

Estimate from time series — I



Estimate from time series — II



University of Portsmouth

Summary — 2nd part

- noise causes early escape near tipping points
- probability depends on type of bifurcation and normal form parameters only
  ⇒study noise effects in normal form

- propensity to escape early can be estimated from time series
- strong non-normality of predicted tipping & escape times

⇒ **[JMTT, JS]** on arxiv

University of Portsmouth