# Introduction

Aim:

- To simulate sub-daily rainfall sequences under scenarios of climate change, using the random parameter Bartlett-Lewis Rectangular Pulse Model (BLRPM).

Approach:

- Estimate future BLRPM parameters using future daily and sub-daily rainfall properties.
- Estimate these future rainfall properties by using:
  - Numerical climate model output
  - Statistical models

# Overview

**Monthly, coarse scale**

Deterministic climate models produce monthly atmospheric sequences.

**Monthly, coarse scale → daily, single location**

Generalised Linear Models (GLMs) simulate non-stationary daily rainfall sequences conditional on coarse scale monthly atmospheric sequences.

**Daily, single location → sub-daily, single location**

Scaling relationships estimate sub-daily rainfall properties conditional on simulated daily rainfall properties.

**Sub-daily, single location**

BLRPM simulates sub-daily rainfall.

# Simulating rainfall via statistical downscaling

## Assumptions:

- Distribution of local-scale rainfall is conditional on atmospheric structure.
- The observed relationships remain valid under altered climatic conditions.
- The relevant aspects of atmospheric structure, and its change in response to greenhouse gas forcing, are realistically represented by the climate models at the temporal and spatial scales used.

# Data used in DEFRA project

- Hourly rain-gauge data from several sites across the UK (1961-2000).
- Concentrate on Heathrow airport here.
- Monthly mean temperature, sea-level pressure and relative humidity from:
  - ‘Observed’ atmospheric data (NCEP reanalysis) at a resolution of 2.5° latitude by 3.75° longitude (1961-2000).
  - Four General Circulation Models (GCMs) and three Regional Climate Models (RCMs) for 1961-1990 and 2071-2100. Future climate model runs forced using the A2 emissions scenario.

# Generalised Linear Models for daily rainfall (1)

Daily rainfall modelled in two stages:

1. Rainfall occurrence modelled using logistic regression. Let $p_t$ denote the probability of rain on day $t$ and let $\mathbf{x}_t$ denote a corresponding predictor vector. Then $\ln(p_t/(1-p_t)) = \mathbf{x}_t'\beta$.

2. Wet day rainfall amounts modelled using a gamma distribution with common shape parameter. Conditional on predictor vector $\mathbf{z}_t$, if day $t$ is wet the mean rainfall is $m_t$, with $\ln(m_t) = \mathbf{z}_t'\alpha$.

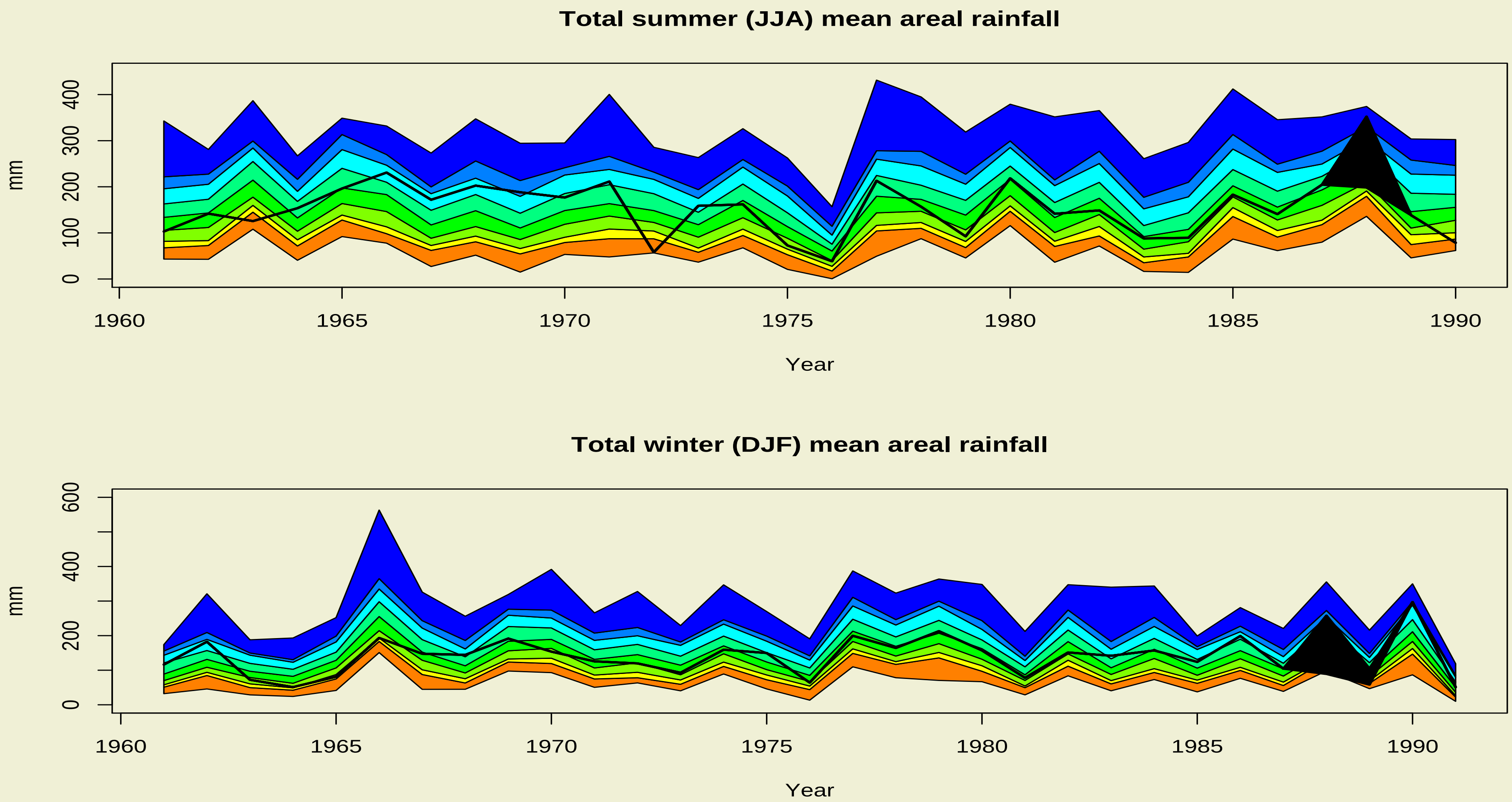
# Generalised Linear Models for daily rainfall (2)

Fit GLMs using standardised monthly NCEP atmospheric data, 1961-1990. Simulate GLMs conditional on standardised monthly GCM and RCM outputs, 2071-2100.

Predictors:

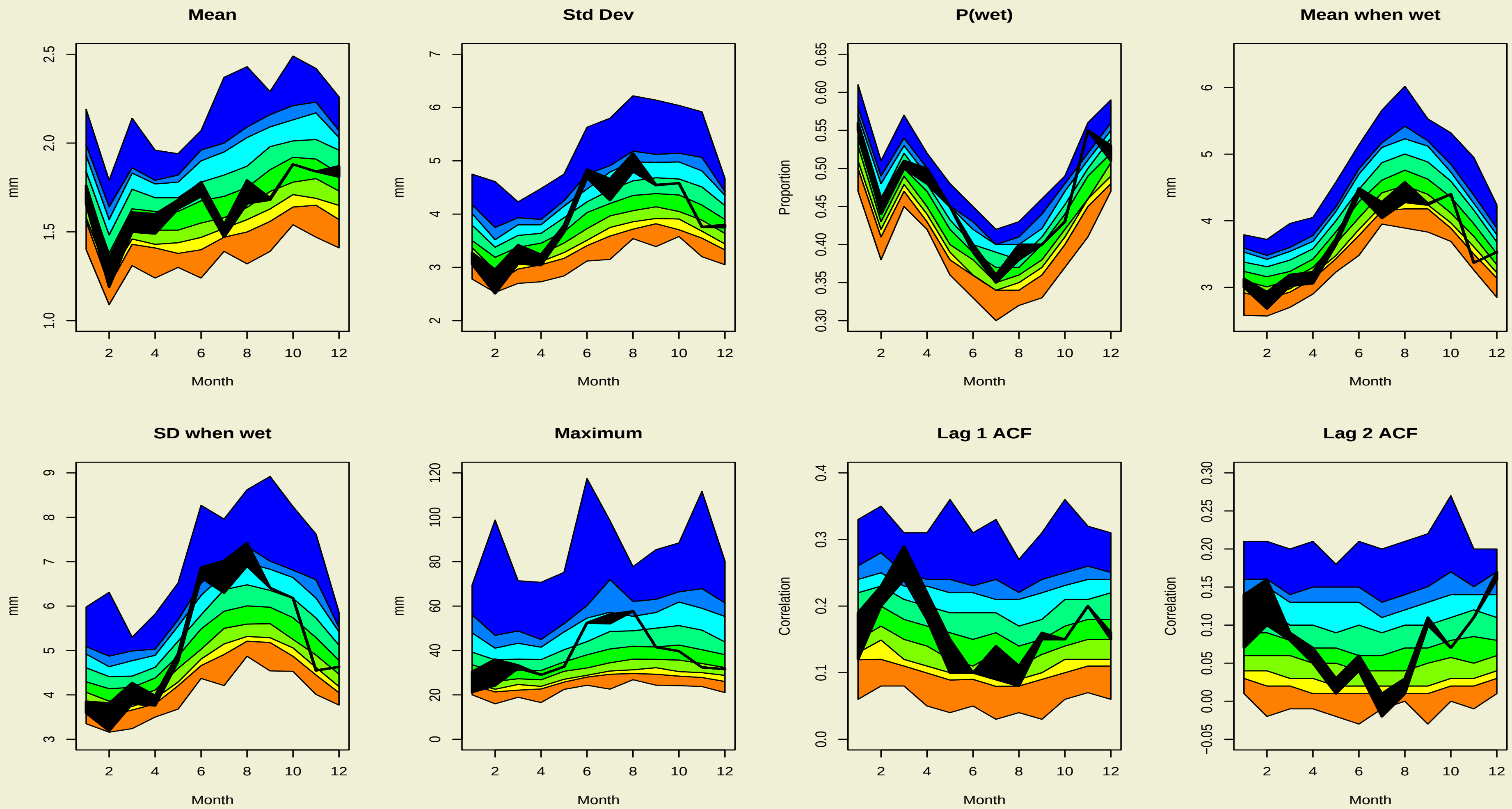
- Seasonality.
- Previous days' rainfall information.
- Atmospheric variables (weighted average of local grid squares).
- Interactions, for example:
  - The dependence on previous days varies over the year.
  - Expect seasonality to alter under climate change.

# GLMs: Model performance 1961-1990, Heathrow



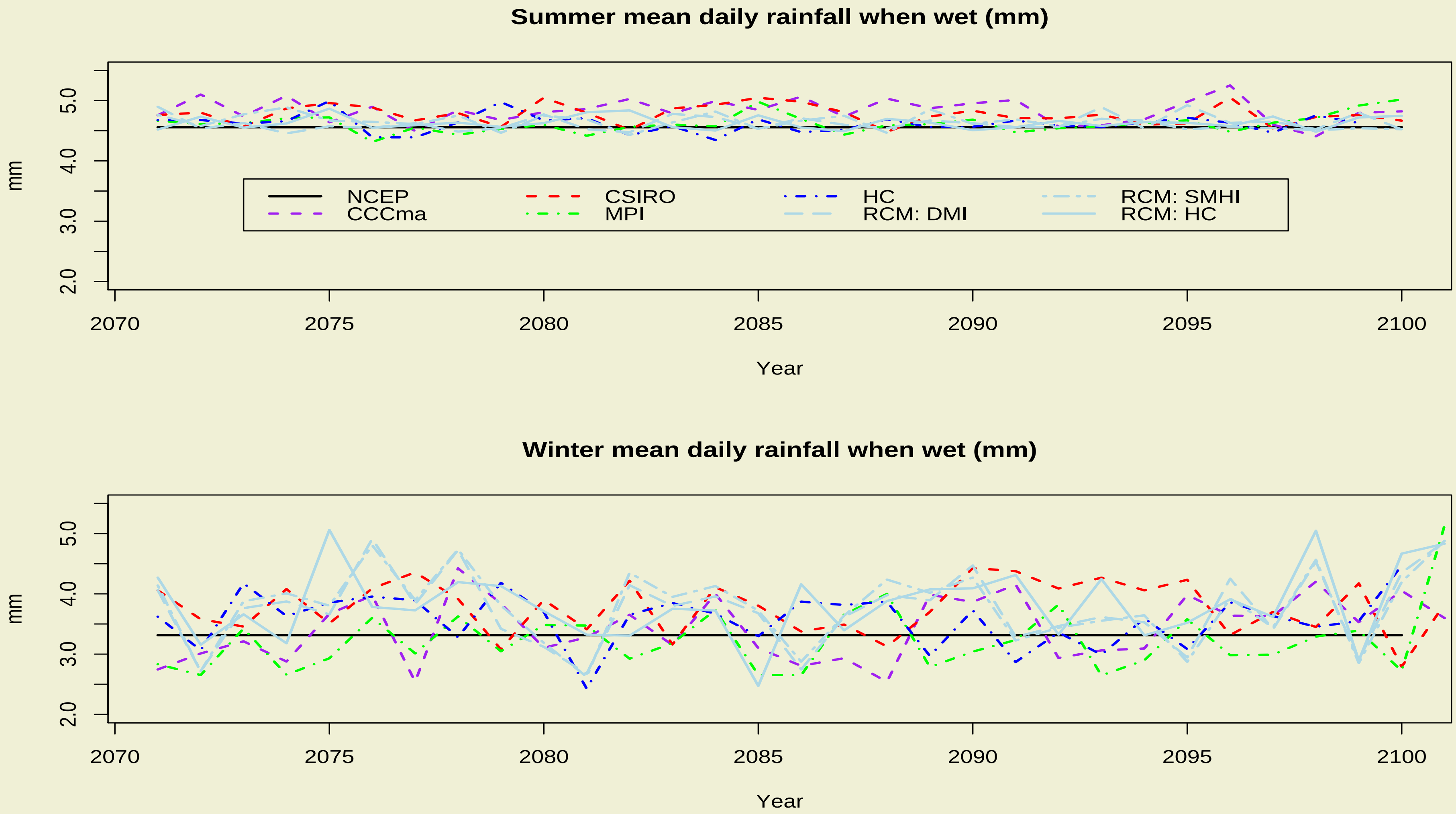

GLM simulated distributions of seasonal rainfall at Heathrow. The bands correspond to the 5th, 10th, 25th, 50th, 75th, 90th, and 95th percentiles and the thick line shows the observed values.

# GLMs: Model performance 1961-1990, Heathrow



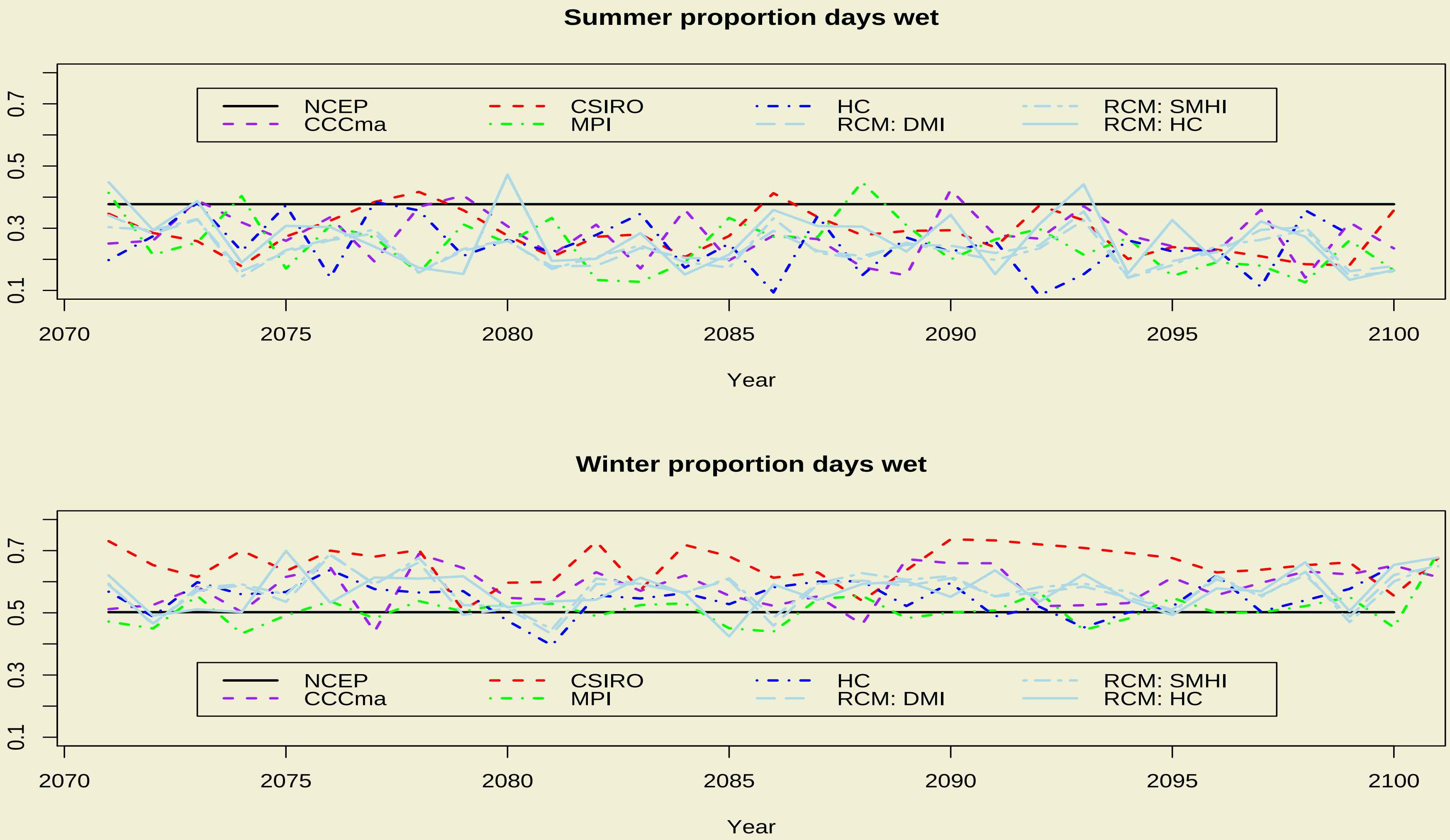

GLM simulated properties of rainfall at Heathrow by month. The bands correspond to the 5th, 10th, 25th, 50th, 75th, 90th, and 95th percentiles and the thick line shows the observed values.

# GLMs: Model simulations 2071-2100, Heathrow



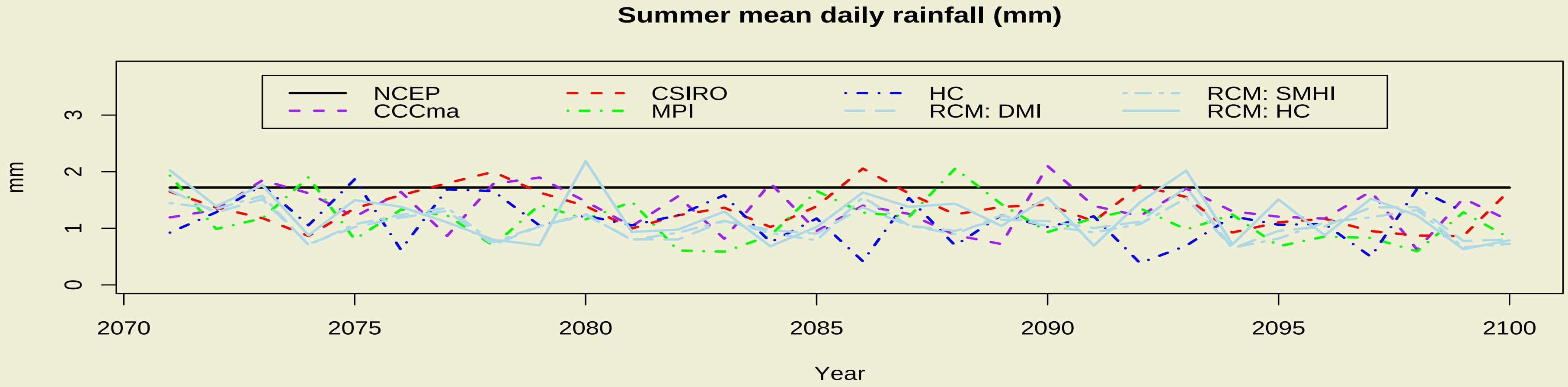

Simulated mean daily rainfall when wet at Heathrow. The black line corresponds to seasonal mean daily rainfall when wet simulated for the 1961-1990 period, conditional on NCEP atmospherics.

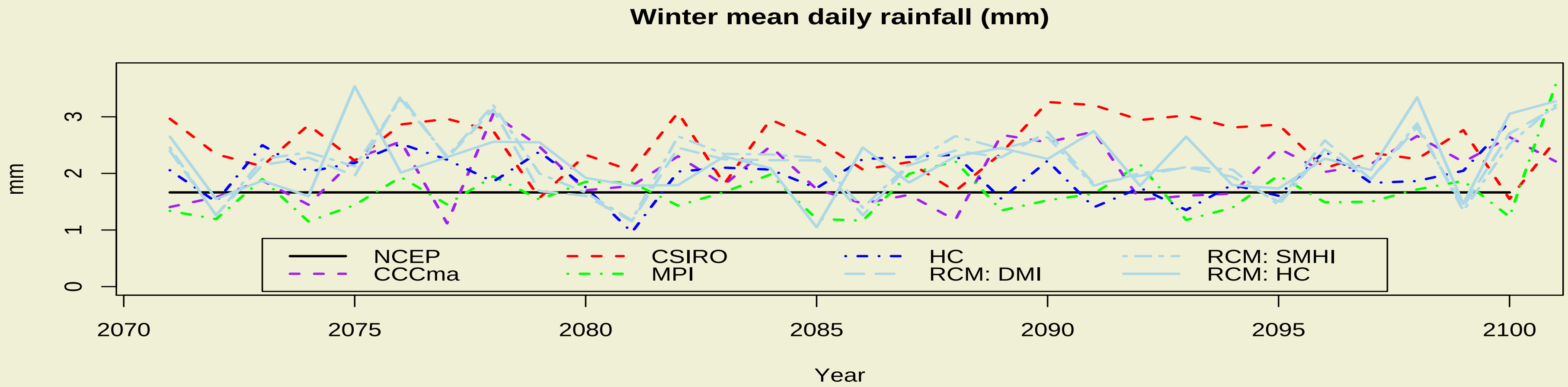
# GLMs: Model simulations 2071-2100, Heathrow




Simulated proportion of wet days at Heathrow. The black line corresponds to seasonal proportion of wet days simulated for the 1961-1990 period, conditional on NCEP atmospherics.

# GLMs: Model simulations 2071-2100, Heathrow




Simulated mean daily rainfall at Heathrow. The black line corresponds to seasonal mean daily rainfall simulated for the 1961-1990 period, conditional on NCEP atmospherics.

# GLMs: Summary

Results:

- Future mean daily rainfall when wet will increase in summer and winter, with respect to 1961-1990.
- Future mean daily rainfall and proportion of wet days will decrease in summer and increase in winter, with respect to 1961-1990 (as will daily rainfall variance).
- There are significant differences between rainfall properties obtained by downscaling different GCMs/RCMs.

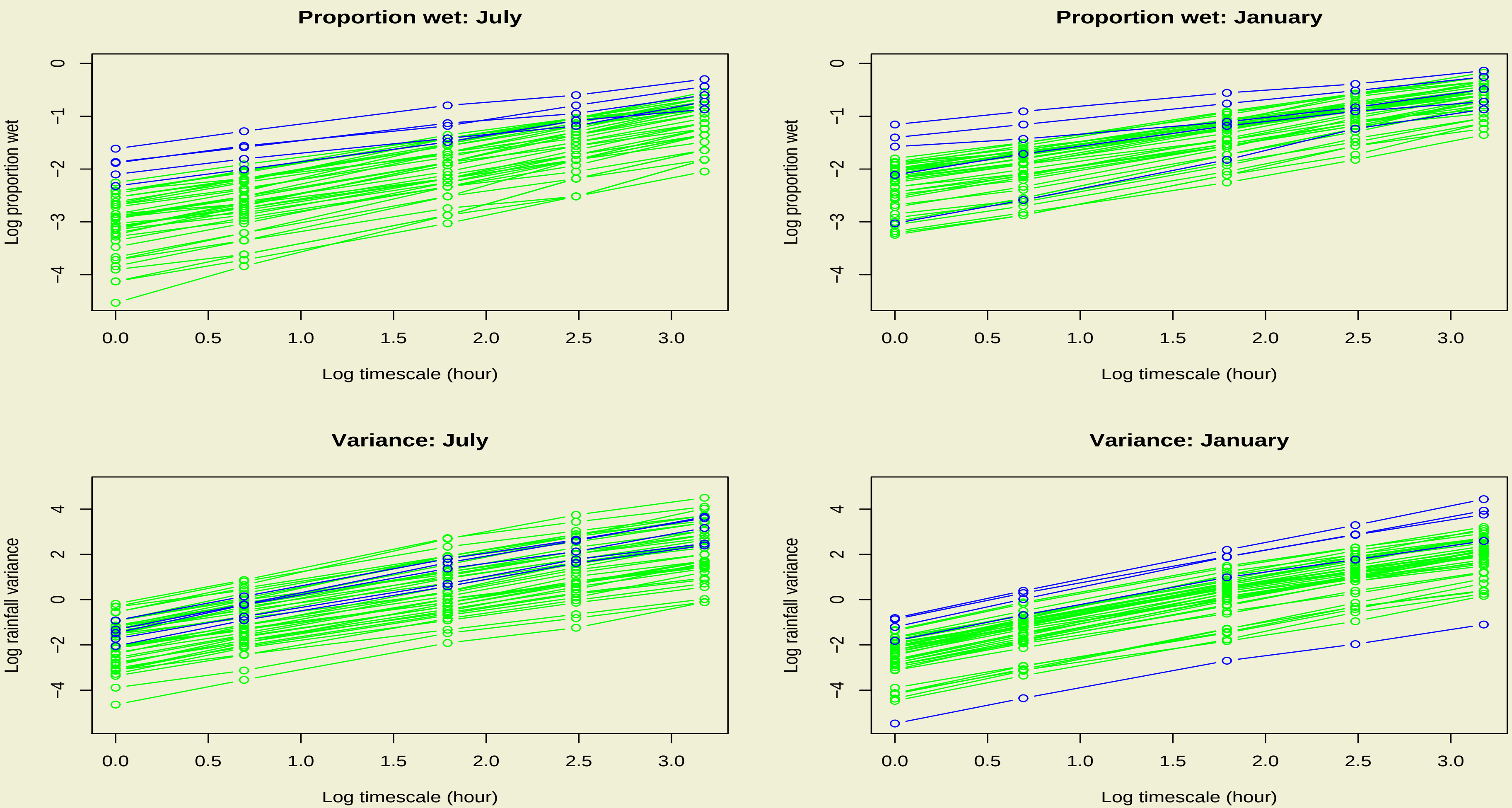
Implications:

- GLMs should be applied to multiple realisations of atmospheric predictors.
- Realisations must accommodate differences between climate models.

# Story so far...

- Require daily and sub-daily rainfall properties under scenarios of climate change in order to estimate BLRPM parameters.
- Daily properties can be simulated from the GLMs. Therefore a methodology to estimate the sub-daily properties conditional on the daily properties is required.
- Any relationship between daily and sub-daily properties in the historical record must still hold under changed climatic conditions. Must therefore be valid under a range of present day conditions.

# Scaling relationships (1)




Properties at Heathrow (1961-2000, green lines) and Malham (1996-2000, blue lines) in January and July.

# Scaling relationships (2)

Let $T_{hmys}$ be either

- observed proportion of wet $h$-hour intervals ($T^{(p)}_{hmys}$)
- observed variance of $h$-hour intervals ($T^{(v)}_{hmys}$)

for site $s$, month $m$ and year $y$. Let $\tau_{hms} = \mathrm{E}[T_{hms}]$.

Model:

$$\ln(T_{hmys}) = \alpha_{mys} + \ln(h)\{\mathbf{x}^{(1)}_{mys}\beta_1\} + (\ln(h))^2\{\mathbf{x}^{(2)}_{mys}\beta_2\} + \varepsilon_{hmys}$$

$\Longrightarrow$

$$\begin{aligned}\ln(\tau_{hmys}) \quad = \quad & \ln(\tau_{24,mys}) + (\ln(h) - \ln(24))\{\mathbf{x}^{(1)}_{mys}\beta^{(1)}\} + \\ & ((\ln(h))^2 - (\ln(24))^2)\{\mathbf{x}^{(2)}_{mys}\beta^{(2)}\}\end{aligned}$$

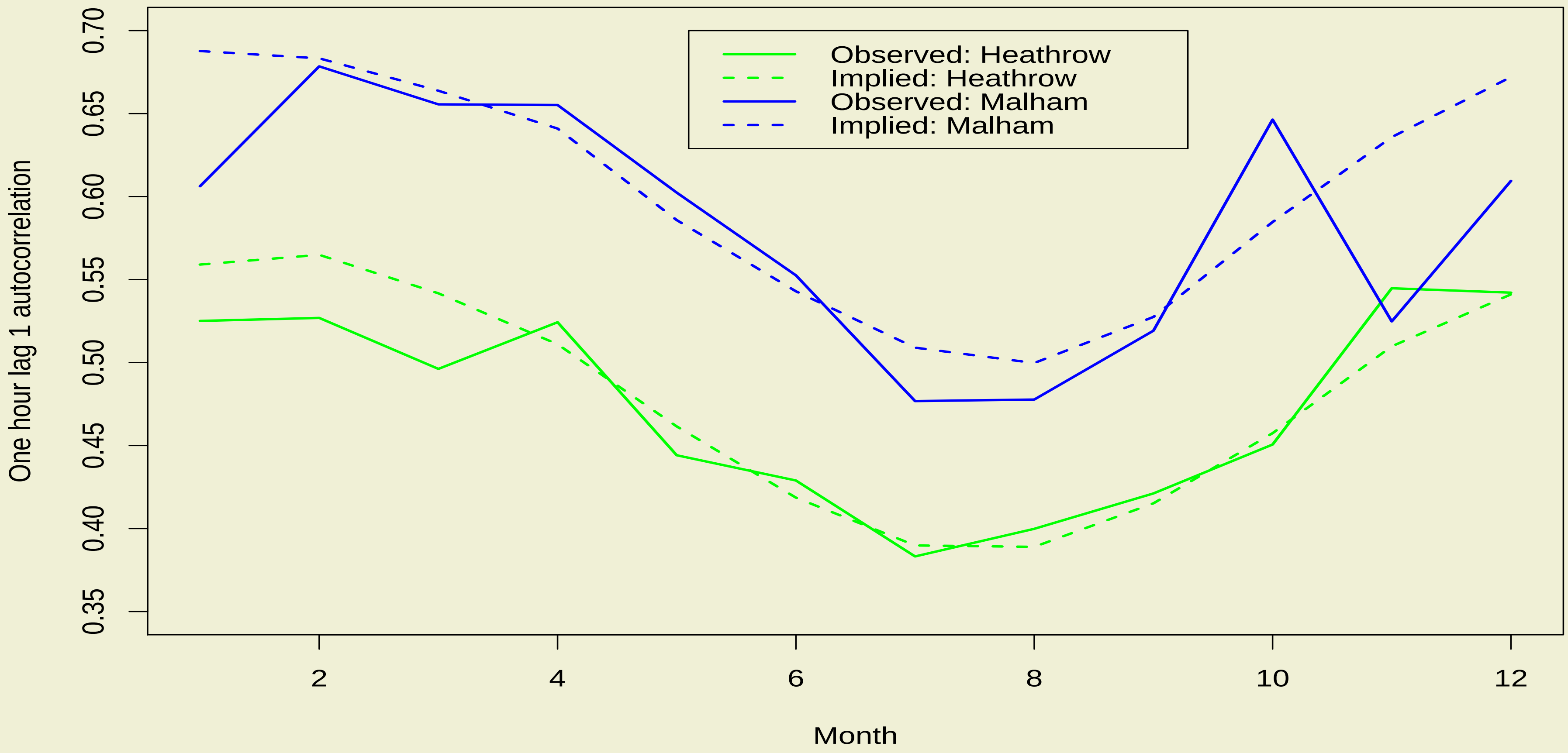
# Scaling relationships: Variance

$$\ln(T_{hmys}^{(v)}) = \alpha_{mys} + \beta_1 \ln(h) + \beta_2 \ln(h)\text{Temperature}_{mys} + \beta_3 \ln(h)\text{Altitude}_s + \beta_4 \ln(h)\text{Northing}_s + \beta_5 (\ln(h))^2 + \beta_6 (\ln(h))^2 \text{Temperature}_{mys} + \varepsilon_{hmys}$$

| Parameter | Estimate | 95% Confidence Interval |
|---|---|---|
| $\beta_1$ | 1.591 | (1.577,1.606) |
| $\beta_2$ | -0.079 | (-0.066,-0.092) |
| $\beta_3$ | 0.261 | (0.216,0.307) |
| $\beta_4$ | 0.009 | (0.005,0.012) |
| $\beta_5$ | -0.069 | (-0.066,-0.073) |
| $\beta_6$ | 0.012 | (0.008,0.016) |

$\beta_3$ is effect per 1km altitude. $\beta_4$ is effect per 100km north.

# Implied autocorrelation from variance scaling relationship




One hour lag-1 autocorrelation at Heathrow (1961-2000) and Malham (1996-2000).

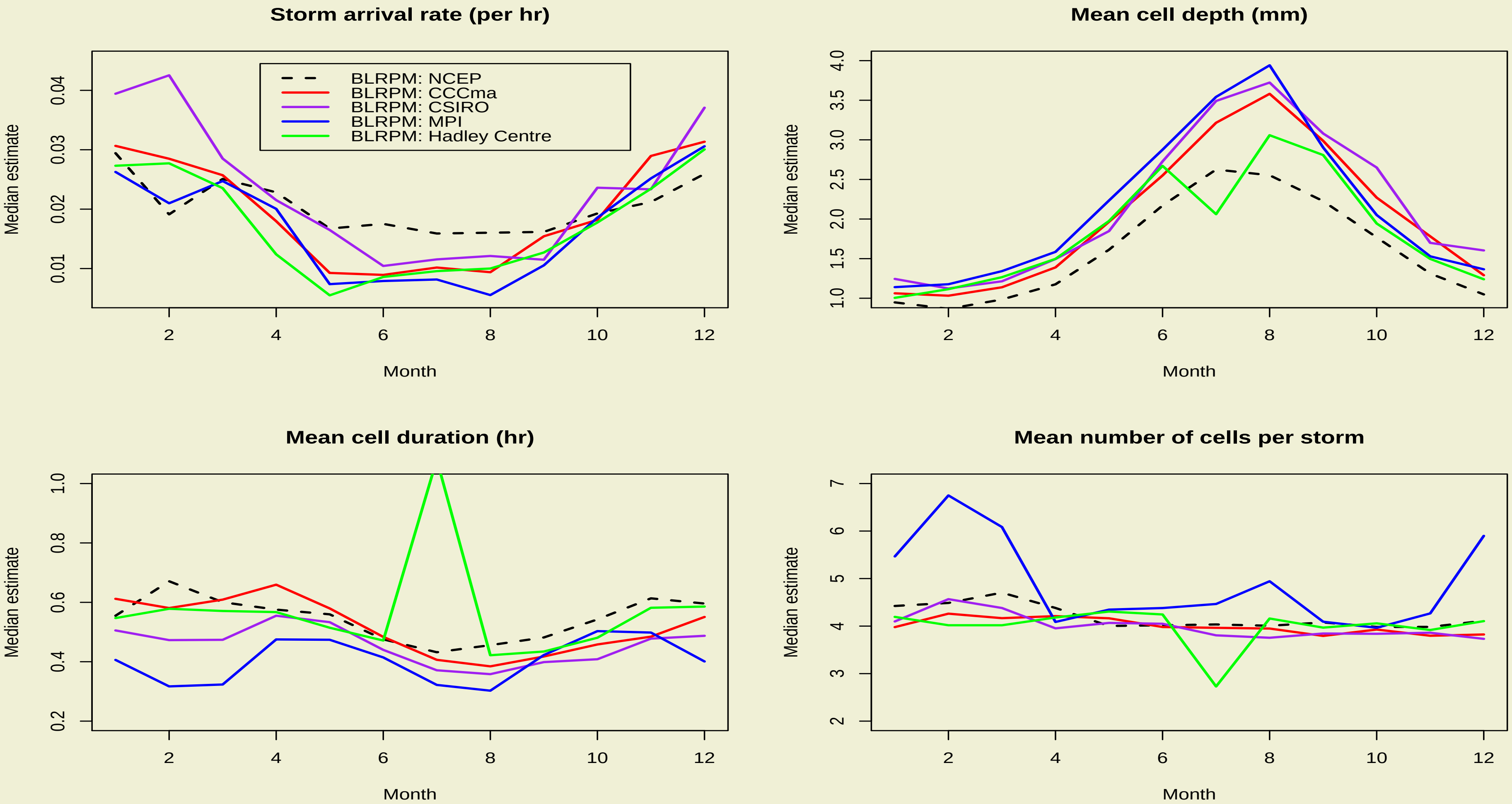
Lag-1 autocorrelation given by $\frac{\tau^{(v)}_{(2h)mys}}{2\tau^{(v)}_{(h)mys}} - 1$.

# Point Process Model

**The Barlett-Lewis Rectangular Pulse Model**:

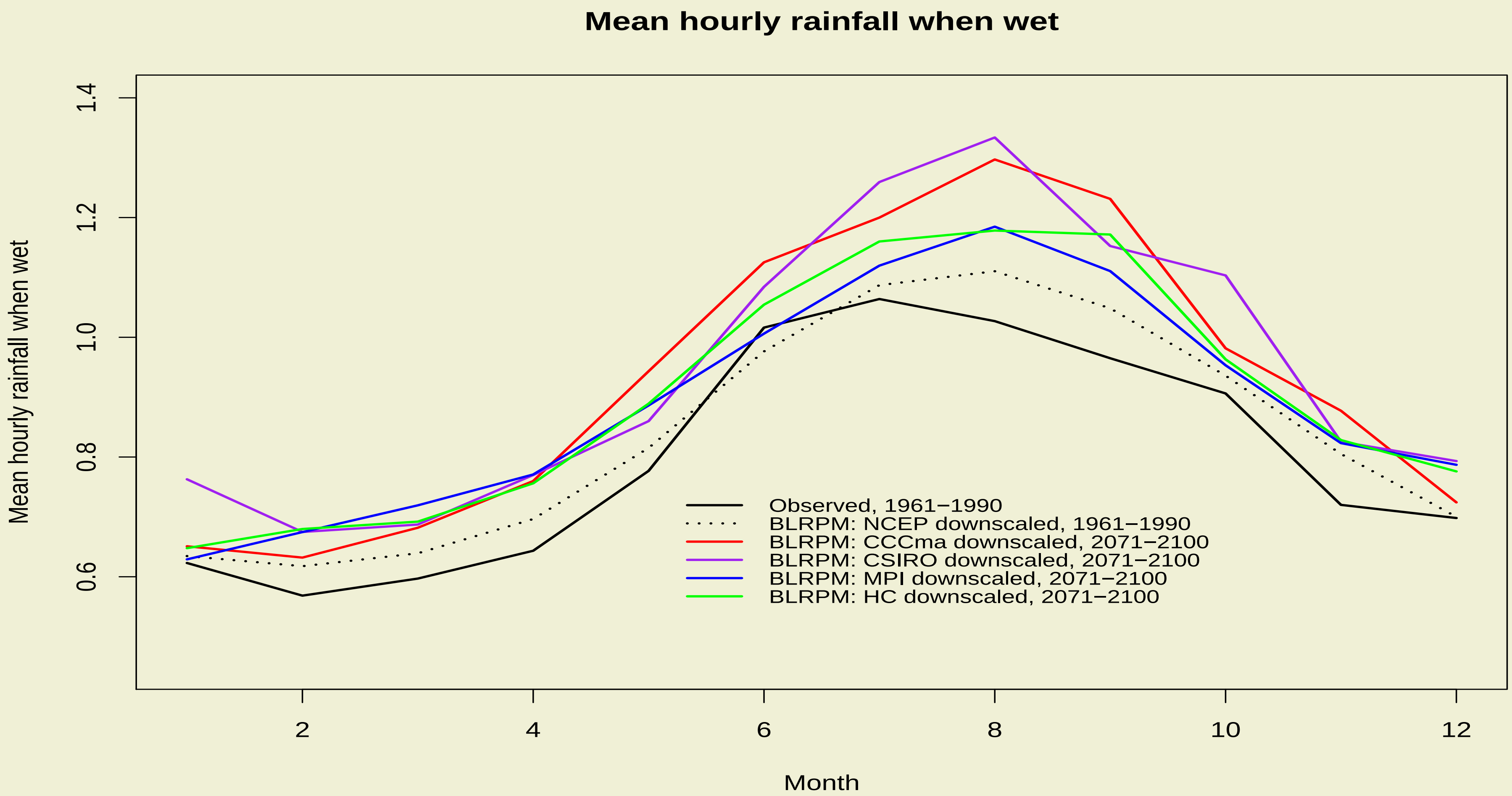
- Storms arrive according to a Poisson process.
- After each storm arrival, there follows a Poisson process of cell arrivals. Each cell duration is independent and exponentially distributed.
- Storms last for a time that is exponentially distributed.

# BLRPM: Parameter estimates



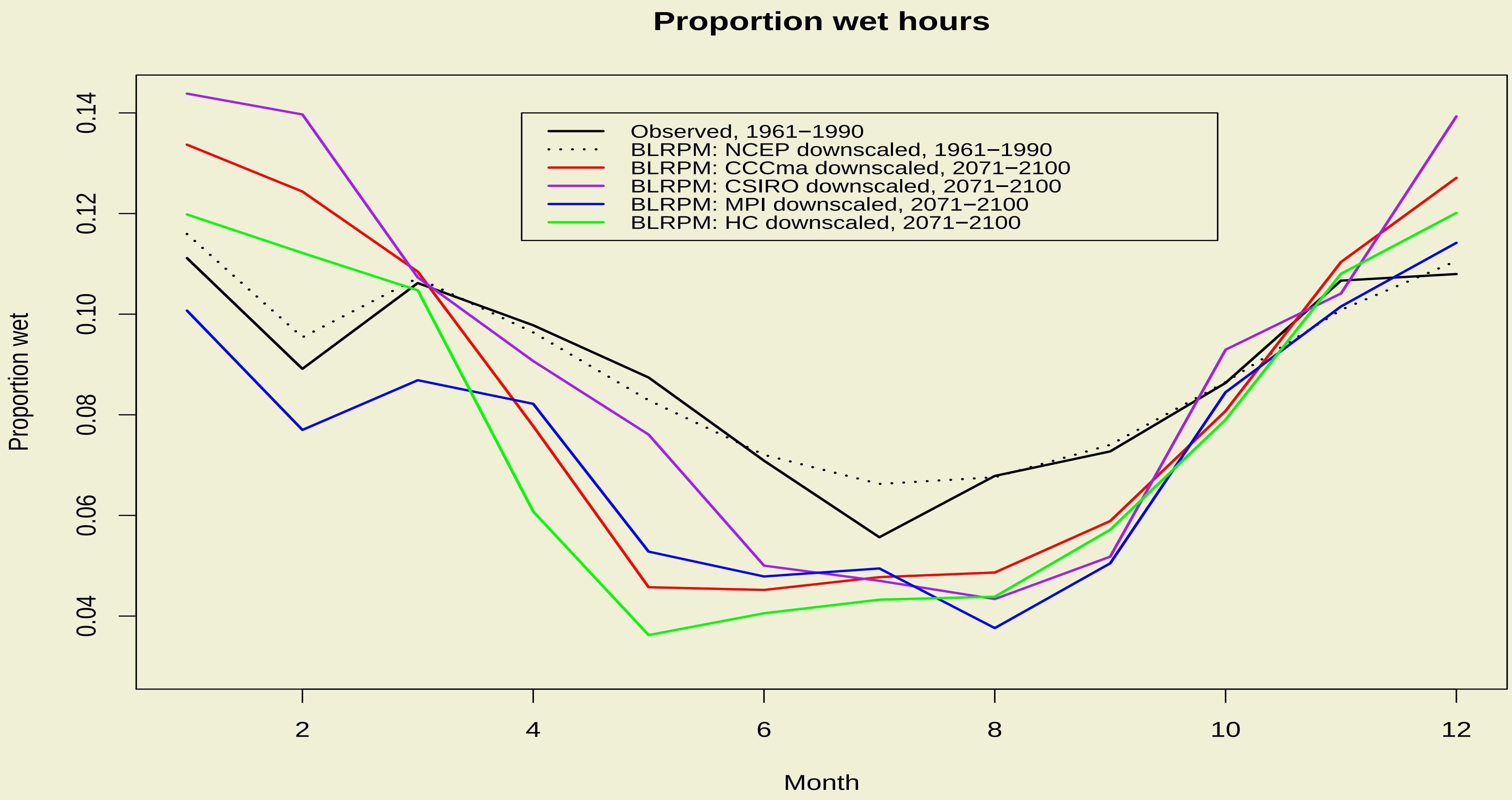

Median BLRPM parameter estimates for 1961-1990 (NCEP) and 2071-2100 (climate models).

# BLRPM: Mean hourly rainfall when wet




Hourly mean rainfall when wet at Heathrow for 1961-1990 and 2071-2100.

# BLRPM: Proportion wet hours




Proportion of wet hours at Heathrow for 1961-1990 and 2071-2100.

# Sub-daily modelling: Summary

- Scaling relationships allow reconstruction of sub-daily rainfall properties.
- Stochastic models provide interpretation of changes in properties (e.g. storm arrival rates).
- Simulations from fitted BLRPMs can be used to drive hydrological rainfall-runoff models.

# Summary

- Methodology to simulate daily or sub-daily sequences.
- Ability to successfully reproduce rainfall properties (beyond those presented here).
- Future rainfall mean, variance and proportion of wet intervals to increase in winter and decrease in summer, at both the daily and hourly level.
- Future daily mean when wet and hourly mean when wet to increase all year round.

## Acknowledgments

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`http://www.ucl.ac.uk/Stats/research/Rainfall/index.html`